Chapter 19

Credit market frictions

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19.1 Introduction

Financial market frictions have long been recognized as an important determinant of macroeconomic dynamics—well before the global financial crisis brought them to the forefront of policy and academic debates. Seminal contributions such as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) incorporated financial frictions into general equilibrium models and helped establish this field as a core area of macroeconomic research. Yet, prior to the Great Recession, mainstream macroeconomic analysis typically abstracted from such frictions. A prevailing view was that, although real-world markets are clearly incomplete, financial imperfections had limited quantitative relevance for business-cycle fluctuations. As a result, it became standard to rely on models with complete markets, which offered tractable analytical solutions. These frameworks incorporated a variety of other frictions—sticky prices and wages, investment adjustment costs, endogenous capital utilization, and labor market matching—but typically left out the financial side of the economy.

The notion that financial frictions were largely irrelevant for macroeconomic analysis is puzzling. One of the most severe economic downturns in modern history—the 1929 Great Depression—was marked by severe disruptions in financial markets, including widespread bank failures and a collapse in credit provision. As discussed in Chapter 1, the Great Depression profoundly influenced the evolution of macroeconomic thought. While several influential scholars emphasized the crucial role of financial markets in explaining the depth and persistence of the Depression (see, for example, Calomiris, 1993 and Bernanke, 2024), financial frictions nevertheless remained peripheral in much of mainstream macroeconomic research. A telling illustration is the volume edited by Kehoe, Prescott, et al. (2002), collecting contributions from leading macroeconomists on the Great Depression: despite the historical context, none of the book chapters explored the role played by financial frictions for understanding the macroeconomic implications of the Great Depression.

The Great Recession in 2007-2009 represents a turning point. The crisis made clear that macroeconomic models constructed on the assumption of frictionless financial markets were missing important elements for understanding the dynamics of the economy. These models needed to be extended to have a more prominent role played by the financial sector. The goal of this chapter is to illustrate an approach to incorporating them.

Given the introductory nature of the chapter, we start with the workhorse macroeconomic framework: the neoclassical growth model. In previous chapters, we analyzed this model under the assumption of complete markets. Here, we relax this assumption and introduce a particular form of financial frictions. While this represents only a specific friction, some of the basic properties illustrated in this chapter are shared by a broad class of models used in the literature. Before describing the formal setup, we first provide empirical motivation for why the joint analysis of financial and real markets deserves attention for understanding macroeconomic dynamics.

19.2 Financial and real markets

Why should macroeconomists pay attention to the joint dynamics of financial and real markets? Besides the anecdotal observation that financial markets could have played some role in specific episodes of booms and busts around the world, a more systematic observation is that credit flows are highly pro-cyclical. The pro-cyclicality is evident not only at the business cycle frequency but also, and perhaps more importantly, over the medium term.

The top panel of Figure 19.1 shows that changes in U.S. credit market liabilities move closely with the U.S. cycle. In particular, debt growth drops significantly during recessions. There are exceptions. For example, credit in the household sector did not contract in the 2001 recession, contrary to business debt. The drop in credit was especially large in 2008 during the Great Recession. The procyclicality of corporate debt has also been shown in many studies using micro data from publicly traded firms.[1] We can also see the cyclical properties of financial markets from more direct indicators of credit tightening based on survey data. The bottom section of Figure 19.1 plots the net percentage of senior bank officers reporting tightening of credit standards for commercial and industrial loans, and for credit cards in the United States. A higher credit standard index indicates that it is more difficult to get a loan from a bank. The figure shows that U.S. banks tighten their credit standards during recessions.

Other indicators of credit tightening such as credit spreads—the interest rate differential on corporate bonds issued by companies with weak credit ratings over the interest paid by government bonds—convey a similar message. As shown in the top panel of Figure 19.2, interest rate spreads tend to increase right before a recession. This suggests a deterioration of the credit capacity of certain businesses, making it more difficult for them to borrow.[2] The bottom panel of Figure 19.2 plots the flow of new debt in the U.S. private sector together with the U.S. unemployment rate. The graph illustrates the strong negative co-movement between the growth of debt and unemployment. This is another illustration of the strong linkage between the real and financial sectors.

Why is the co-movement in real and financial variables relevant? In a world in which markets are complete, which is the case in the standard neoclassical model, the financial structure of households and firms would not necessarily follow a pro-cyclical pattern. For example, since in a recession there are more unemployed workers, the household sector could borrow more to fund the consumption of unemployed workers. This would lead to a

[1] See Covas and denHaan (2011) and Begenau and Salomao (2019).

[2] See also Gilchrist, Yankov, and Zakrajsek (2009).

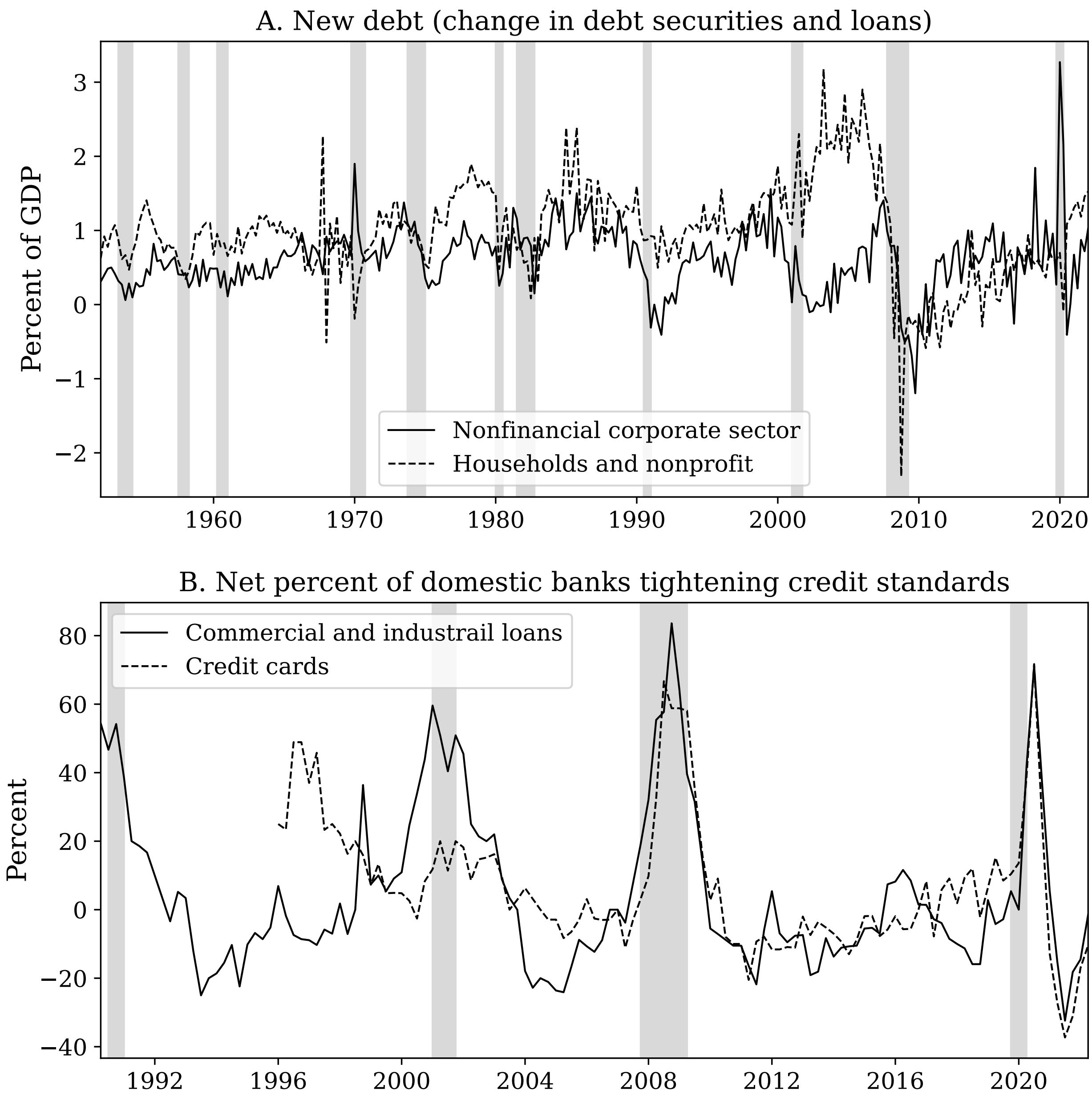

Figure 19.1: **Panel A**: Change in the volume of U.S. credit market instruments in the household and business sectors as a percentage of GDP. **Panel B**: Index of credit tightening in commercial and industrial loans and in credit cards.

counter-cyclical flow of new debt. The fact that credit flows are pro-cyclical and the index of tightening standards is counter-cyclical suggests that the complete-market paradigm has limitations in capturing the joint dynamics of real and financial variables. This is especially true for the index of credit tightening: if markets were complete, there is no reason for lenders to change their ‘credit standards’ over the business cycle.

However, the fact that there is a strong co-movement between financial and real flows—as shown in Figure 19.1 and the bottom panel of Figure 19.2—does not in itself establish

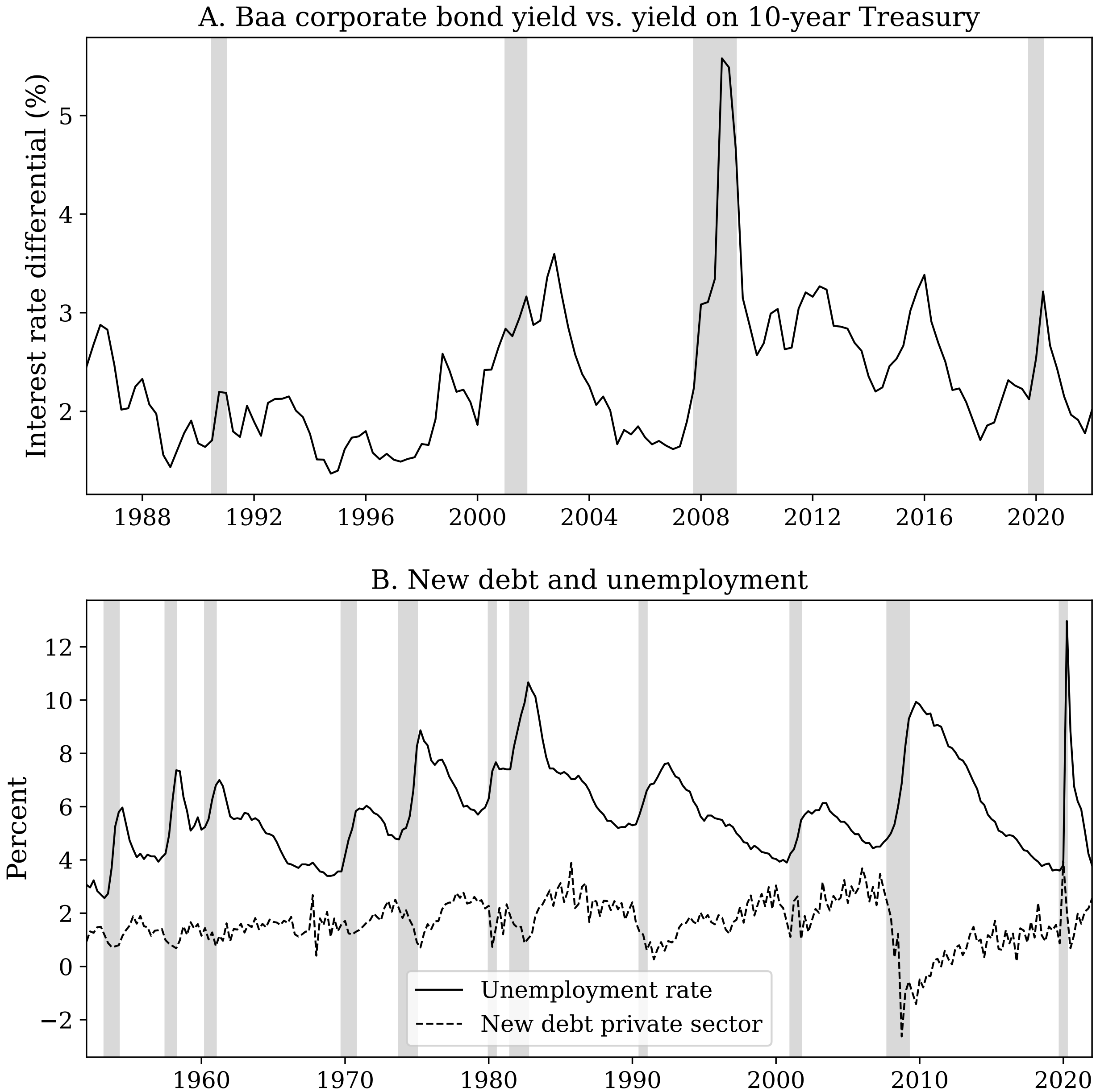

Figure 19.2: **Panel A**: Interest rate differential between U.S. corporate bonds issued by companies with a Baa credit rating and the yield on 10-year Treasury notes. **Panel B**: Change in the volume of credit market instruments in the U.S. private sector (households and businesses) as a percentage of GDP and U.S. unemployment rate as a percentage of the labor force.

causality: does lower credit growth cause recessions or do recessions cause lower credit growth? Conceptually, we could have three possible scenarios:

1. *Real activity causes movements in financial flows.* It is possible that the contraction in consumption and investment expenditures, as well as employment, is the direct response to changes in real factors such as productivity. In this case, borrowers cut

their debt simply because they need fewer funds to conduct economic transactions and to finance their expenditures. If this were the only linkage between real and financial flows, the explicit modeling of the financial sector would be of limited relevance for understanding movements in real economic activity. The neoclassical model is built on this assumption.

2. *Propagation.* A second possibility is that nonfinancial factors, such as changes in productivity, are the initial driving forces behind movements in economic activity. However, financial factors affect how these initial changes propagate to the real sector of the economy. For example, as investment and employment both fall in response to a decline in productivity, the credit capacity of borrowers also deteriorates. This could happen, for instance, if the fall in investment generates a fall in the market value of assets that are used as collateral. The deterioration in accessible credit forces borrowers to reduce investment and hiring more than they would have in the absence of the credit contraction. Thus, financial frictions could *amplify* the macroeconomic impact of nonfinancial shocks.

   Although in this example financial frictions amplify the shock, they could also dampen it. For example, an improvement in technology stimulates investment. However, the presence of financial frictions could limit this growth, as the credit required to finance the desired investment boom might be unavailable. In general, whether financial frictions act as amplification or dampening mechanisms, it is plausible that they impact—at least to some extent—the propagation of nonfinancial shocks. The key question, then, is whether their impact is quantitatively significant.

3. *Financial shocks.* A third possibility is that the initial disruption or shock arises directly in the financial sector. After hitting the financial sector, the shock propagates to the real side of the economy. For example, a disruption in financial markets could make it more difficult to transfer funds from lenders to borrowers causing a contraction in consumption and investment spending and leading to a decline in employment. This type of disruption is now commonly referred to as a 'credit' or a 'financial' shock. In reduced form, a financial shock can be modeled as a change in the parameter that governs the severity of financial frictions in the model.

Prior to the Great Recession, a large majority of studies in the macro-finance literature focused on the second possibility, that is, on the role of financial frictions for the 'propagation' of nonfinancial shocks.[3] The main idea was that financial frictions could make the real impact of a shock bigger (amplification) or smaller (dampening). However, financial frictions were not the initial 'cause' of macroeconomic expansions and contractions: something else has to happen first in the nonfinancial sector. The analysis of financial shocks as a 'source' of macroeconomic fluctuations has received more attention since the 2008 financial crisis and the Great Recession.

This chapter presents parsimonious ways to formalize financial frictions and illustrates how they shape the transmission of both nonfinancial and financial shocks to the rest of

[3] This was the case for Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) and the related literature.

the economy. We do not discuss extensively the first case because, as observed above, if real shocks are the dominant drivers of macroeconomic fluctuations, explicitly modeling the financial sector adds limited insight into the dynamics of the real economy.

19.3 Modeling financial frictions

Financial frictions arise when some financial assets or claims cannot be traded. In the context of an Arrow-Debreu economy, they correspond to situations in which markets for certain contingencies are missing and those trades cannot occur. This limits the agents' ability to shift spending across time or states of nature. The absence of trading markets becomes relevant only if agents are heterogeneous in some dimension that induces a reason to trade. Thus, all models with relevant financial frictions share two features:

1. *Missing markets*: Markets for trading certain financial claims are missing.

2. *Heterogeneity*: Agents are heterogeneous in some meaningful dimension.

These two characteristics are necessary for financial frictions to be relevant, but they are not sufficient. The absence of some markets could be irrelevant if, in equilibrium, agents would not have chosen to trade in them anyway. Hence, market incompleteness and heterogeneity must interact for financial frictions to matter for the aggregate economy. The literature has explored this interaction through various forms of market incompleteness and heterogeneity.

19.3.1 Missing markets

To illustrate the role of missing markets, it is useful to begin with a counter-example: the Modigliani and Miller (1958) theorem. The principle states that, in an economy with complete markets, the source of funding—such as debt versus equity—has no effect on real decisions. In particular, whether an agent finances investment through debt or equity does not affect the optimal level of investment; the agent is indifferent between the various sources of funding. In contrast, when financial frictions are present, this indifference breaks down: agents' financial choices affect their real decisions. To analyze such environments, we must therefore depart from the frictionless benchmark.

Modigliani-Miller principle. To illustrate the Modigliani-Miller principle, we use a version of the complete market model studied in Chapter 7. A firm purchases capital k at time 0, at price 1 (one unit of capital at time 0 is worth 1 unit of time-0 consumption goods). Then in period 1, the state i (shock) is realized, after which the firm hires labor ℓ_i at price $p_i w_i$. Here p_i is the Arrow-Debreu price (consumption goods paid in period 0 for the delivery of one consumption good in period 1) and w_i is the price of one unit of labor in terms of consumption goods paid in period 1. Thus, $p_i w_i$ is the wage in terms of consumption goods in period 0. The firm produces $F(z_i, k, \ell_i)$ at date 1 and each unit of output is worth p_i units of consumption goods in period 0. Finally, the firm sells the undepreciated capital $(1 - \delta)k$, each unit worth p_i units of period-0 consumption.

The firm solves the problem

$$\max_{k,\{\ell_i\}_i} \Big\{ -k + \sum_i p_i \Big[F(z_i, k, \ell_i) - \ell_i w_i + (1-\delta)k \Big] \Big\}. \tag{19.1}$$

This problem does not specify how the firm finances the purchase of capital k in period 0. So let's assume that the firm has two choices: debt, denoted by b, or equity, denoted by e. In the first case the firm will borrow while in the second the firm sells shares to future dividend payments. Given these two sources of funding, we have that $k = b + e$.

Denote by R the risk-free gross interest rate paid on the debt. Then, the value of dividends in terms of period-0 consumption paid by the firm in period 1 at state i are $d_i = p_i[F(z_i, k, \ell_i) - \ell_i w_i + (1-\delta)k - Rb]$. Because the firm maximizes the value of equity, the shareholders' optimization problem is

$$\max_{k,\{\ell_i\}_i} \Big\{ -(k-b) + \sum_i p_i \Big[F(z_i, k, \ell_i) - \ell_i w_i + (1-\delta)k - Rb \Big] \Big\}. \tag{19.2}$$

Shareholders contribute $e = k - b$ at time 0 and receive the dividends d_i in period 1.

As we have seen in Chapter 7, in a complete markets equilibrium, prices satisfy $\sum_i p_i R = 1$. This implies that the variable b cancels out in Problem 19.2 and we get back to the previous Problem 19.1 which does not specify the source of funding. Hence, in a frictionless (complete-markets) environment, the financial structure—debt versus equity—does not affect real decisions such as investment or labor demand. The firm is indifferent among financing methods because all assets are priced so that expected discounted returns are equal. As a result, the choice of the financial structure—b versus e—is indeterminate.

The argument is very general and applies to any set of financial instruments (e.g., short-term vs. long-term debt) as long as they are priced competitively. The argument could also apply with incomplete markets, provided that all instruments are priced consistently with firms' stochastic discount factors. In this chapter, however, we will consider environments in which these conditions fail—markets are incomplete, certain assets are missing, or financial contracts are constrained—so that the Modigliani-Miller neutrality no longer holds and financial frictions become relevant to macroeconomic dynamics.

Modelling missing markets. The approaches used in the literature to formalize missing markets and possibly leading to the violation of the irrelevance principle of Modigliani and Miller, can be divided in two categories: 'exogenous' and 'endogenous'.

1. *Exogenous market incompleteness.* In this category we include models that impose, by assumption, that certain claims or assets cannot be traded. A common assumption is that agents can only save or borrow using non-contingent bonds. In other words, they cannot purchase or issue bonds with payoffs that are contingent on information that will be revealed in the future. In general, the goal of these studies is not to explain why certain assets cannot be traded but to understand the consequences of missing markets.

   These studies start from the observation that in the real world a large volume of financing transactions is in the form of borrowing and lending with fixed repayment (standard debt contracts). Of course, there are also financial contracts with payouts contingent on future events. However, the volume of these contracts is smaller than the theory would predict. The modeling assumption that borrowing can be done only with non-contingent debt is a pragmatic approximation to the complexity of the actual economy. Another common assumption is that there is a limit to the debt that an individual agent can take (an exogenous borrowing constraint). The limit is not necessarily fixed and could depend on the characteristics of the borrower. However, there is no attempt to motivate with explicit micro-foundations why borrowing is limited.

2. *Endogenous market incompleteness.* Models with endogenous market incompleteness assume that the set of feasible contracts is restricted by incentive considerations. Markets are missing because parties are not willing to engage in certain trades as a result of agency frictions, which typically take one of two forms:

   (a) *Information asymmetry.* In many instances, borrowers have more information than lenders. For example, if repayment is contingent on the performance of the business and that performance depends on 'unobservable' effort from the borrower, the borrower has an incentive to choose lower effort. Since effort is observed only by the borrower, shirking cannot be detected. This creates a moral hazard problem. In some contexts, information asymmetry gives rise to adverse selection: since the riskiness of borrowers cannot be detected by lenders, those applying for loans tend to be riskier borrowers. Adverse selection is commonly viewed as a central friction in insurance markets.

   (b) *Limited enforcement.* Lenders may have the same information as borrowers and can observe whether borrowers deviate from the agreed-upon action—for example, the repayment due after a specific event occurs. However, lenders are unable to force borrowers to fulfill their obligations. If borrowers repay less than initially agreed, lenders have no effective mechanism to enforce the contract.

Although models with limited enforcement are typically easier to characterize analytically than models with information asymmetries, they share a common property: the higher is the borrower's net worth, the greater the amount of (incentive-compatible) external funds that can be raised. This is also a property of many models with 'exogenous' market incompleteness. Hence, for most macroeconomic applications, the precise source of market incompleteness—whether exogenous or endogenous—has limited implications for the results. There are, however, important cases in which this distinction matters. Additionally, an advantage of models where market incompleteness is endogenous, is that they are less vulnerable to the Lucas critique, that is, the fact that certain parameters may not be invariant to policy changes. A simple example is a policy that increases the penalty for debt default. Because of the higher penalty, creditors anticipate that borrowers have less incentive to renege on their debt obligations and, as a result, they are willing to lend more. This effectively relaxes the borrowing constraint, something that would not be captured in a model with exogenous borrowing constraints. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) provide classic examples of endogenous borrowing constraints—the former arising from information asymmetries, the latter from limited enforcement.

19.3.2 Heterogeneity

There are different approaches used in the literature to incorporate heterogeneity. In some models, agents are ex-ante identical but become heterogeneous ex-post due to uninsurable idiosyncratic shocks, as in the Bewley-Huggett-Aiyagari frameworks discussed in Chapter 11. Because market incompleteness generates a high-dimensional distribution of heterogeneity, most applications abstract from aggregate shocks.[4] By contrast, the majority of studies analyzing financial frictions under aggregate uncertainty employ alternative modeling approaches where heterogeneity is more tractable.

A popular approach is to assume that there are only two types of agents that are ex-ante and permanently different in preferences and/or technology. In equilibrium, one type of agents borrows while the other lends. In some cases, heterogeneity is present within a given type of agent. However, the aggregate behavior of each type can still be characterized by that of a representative agent. This is because the model allows for linear aggregation among agents belonging to the same type (but not across different types).[5].

In many models, financial frictions are important because they limit the reallocation of resources from agents that are less productive to agents that are more productive. There is an incentive for those who expect to be more productive in the future to save in order to overcome potential borrowing constraints. Over time, these agents may accumulate enough wealth so that they are no longer dependent on external funding. To maintain the significance of financial frictions in the long run, further assumptions are typically needed. The box provides a non-exhaustive list of assumptions made by various studies in the literature.

Assumptions that prevent self financing

Finite life span. Some models assume that borrowers are finitely lived. Examples include models with overlapping generations where newborn agents have no initial wealth. Over time, they accumulate wealth and become unconstrained. However, since there are newly-born agents every period, at any point in time there is always a subset of agents that face binding financial constraints. A similar mechanism arises in models with an industry dynamic structure where exiting firms are replaced by new entrant firms.

Different discounting. Another popular approach is to assume that agents are infinitely lived but some of them discount the future more than others. More impatient agents end up being the borrowers while patient agents lend. For impatient agents, the cost of external financing—the interest rate—is lower than their inter-temporal discount rate. Therefore, they do not save enough to reach a point at which the borrowing constraint is no longer binding in the long run. This implies that in every period some agents are always constrained.

[4] Only a few notable exceptions include Krusell and Smith (1998), Cooley, Marimon, and Quadrini (2004), Guerrieri and Lorenzoni (2010), Khan and Thomas (2011) and Arellano, Bai, and Kehoe (2019).

[5] Carlstrom and Fuerst (1995) and Bernanke, Gertler, and Gilchrist (1999) are examples of this approach.

Tax benefits. A comparable result can also be generated by introducing tax advantages to debt financing. For example, the tax deductibility of interest payments from corporate earnings generates a preference for debt financing over equity, leading to higher leverage. In this case the two types of agents are, typically, households (who do not get a tax benefit from debt) and firms (who benefit from the debt shield). This assumption is especially popular in structural finance with various applications to macroeconomics.

Convenience yield. An assumption that is becoming popular in the macro-finance literature is that debt issued by certain agents provides liquidity services to others. This feature is often referred to as a 'convenience yield.' The idea is that some forms of debt are safe, can be easily liquidated, and are readily accepted as collateral in emergencies, offering holders special benefits relative to other assets. An equivalent interpretation is that debt provides a 'utility flow' to its holders. Because of this additional benefit, agents are willing to hold it even if the interest rate is below the inter-temporal discount rate. Although the convenience yield is most commonly associated with government bonds, the same logic can be applied to other types of debt.

Wage bargaining. A further assumption considered in the literature is that external financing (debt or outside equity) is preferred to inside financing (entrepreneurial equity) because it affects the bargaining position of firms in the negotiation of wages and/or executive compensation. If the compensation of workers and/or managers is determined through some form of bargaining (such as via labor unions in the case of workers), highly levered firms would be able to negotiate lower wages because existing debt reduces their bargaining surplus. This mechanism introduces an incentive for firms to take on more debt and can leave them financially constrained.

This section has outlined the most popular approaches used in the literature to formalize financial frictions. Of course, it is not possible to illustrate all of them in detail in a single chapter. We will then focus on a particular formalization. However, the most salient properties that will be outlined in the next sections are similar to those characterized in models that adopt a different formalization of financial frictions.

19.4 Adding financial frictions to the neoclassical model

The starting point is the neoclassical growth model studied in previous chapters. In that model, markets are complete and there is only one representative agent. Here we maintain the assumption that there is a representative household that has the same characteristics as in the standard model. However, we place some more structure on the operation of firms. More specifically, we assume that capital accumulation (investment) is chosen by firms instead of households. This implies that firms solve a dynamic problem: the investment made today increases profits in the future.

Households remain the ultimate owners of firms, which operate to serve their interests. However, for analytical convenience, we treat firms as a second type of agent, separate from households. This distinction introduces heterogeneity into the model. As we will see once additional assumptions are introduced, firms borrow from households and households lend to firms in equilibrium.

The objective of firms is to maximize the present value of dividends. Since households own the firms and firms operate on their behalf, the discount factor used by firms is the same as that of the households. Consequently, firms make the same production and investment decisions that households would make if they operated the firms directly.

Before turning to the optimization problems of firms and households, it is useful to first define the discount factor. For a household, a unit increase in consumption at time $t > 0$ has a utility value of $u_1(C_t, L_t)$, where $u_i(\cdot, \cdot)$ represents the derivative with respect to the ith argument. Thus, $u_1(C_t, L_t)$ is the marginal utility of consumption at time t. The discounted value at time zero of that utility is $\beta^t u_1(C_t, L_t)$. We now ask the following question: What should be the increase in consumption at time zero that gives the same flow of utility as one unit increase in consumption at time t? Denote by Δ the marginal increase in consumption at time zero. The extra utility received at time zero from this increase is $\Delta \cdot u_1(C_0, L_0)$, that is, the increase in consumption multiplied by the marginal utility of consumption at time zero. The increase in consumption Δ that provides the same utility as the unitary increase in consumption at time t is determined by solving the condition $\Delta \cdot u_1(C_0, L_0) = \beta^t u_1(C_t, L_t)$. Rearranging we obtain

$$\Delta = \frac{\beta^t u_1(C_t, L_t)}{u_1(C_0, L_0)}.$$

Thus, a unit of consumption at time t is equivalent to $\beta^t u_1(C_t, L_t)/u_1(C_0, L_0)$ units of consumption at time zero. The term $\beta^t u_1(C_t, L_t)/u_1(C_0, L_0)$ is known in the literature as the 'stochastic discount factor.' Because the stochastic discount factor captures how households—the owners of firms—evaluate future payments in the current period, firms use this factor to discount dividends that will be paid in the future. For notational convenience we denote the stochastic discount factor using the variable $m_t = \beta^t u_1(C_t, L_t)/u_1(C_0, L_0)$, a notation often used in the field of finance.

The firm's objective function can be expressed as

$$\mathbb{E} \sum_{t=0}^{\infty} m_t d_t,$$

where d_t are the dividends paid by the firm at time t. The objective of the firm, thus, is to maximize the expected discounted value of dividends using m_t to discount dividends paid in the future.

The next step is to place some structure on the financing decisions of firms. Since capital is accumulated by firms, they need to finance investment. We allow for two forms of financing: equity and non-contingent debt. Equity refers to the households' claims to the dividends of the firm. Non-contingent debt is the amount borrowed from households. If the firm borrows b_{t+1}/R_t at time t, it promises to repay b_{t+1} units at time $t+1$. The variable R_t is the gross interest rate (one plus the interest rate), while its inverse, $1/R_t$, is the price of the debt. The interest rate is determined in general equilibrium to clear the credit market.

So far, even though we added more structure to the neoclassical growth model by assuming that capital is accumulated by firms and funded with debt and equity, this modification is inconsequential for equilibrium allocations. In the absence of additional assumptions, the equilibrium level of firm debt is indeterminate, and the real allocation coincides with that of the standard neoclassical growth model (as shown formally, later, in Proposition 19.1). This result again illustrates the Modigliani-Miller irrelevance principle. To make financial decisions matter for real outcomes, we need to introduce additional frictions. Assumptions (19.4.1)-(19.4.3) capture those frictions and differentiate the model with financial imperfections presented here from the frictionless benchmark.

The first assumption is that debt financing is cheaper than equity financing.

Assumption 19.4.1 *The cost of debt for firms is lower than the cost of equity.*

One way in which we can make this assumption operational by incorporating a tax advantage to debt financing: if the firm pays a gross interest rate R_t, it receives an indirect tax benefit from the tax deductibility of interest payments from corporate profits. The tax benefit is equivalent to a subsidy $\tau > 0$ so that the effective gross interest rate paid by the firm is

$$\tilde{R}_t = R_t/(1+\tau).$$

This assumption has been used by several studies in structural corporate finance (see, for example, Hennessy and Whited, 2007) and macro-finance (see, for example, Jermann and Quadrini, 2012). However, this is not the only way to generate a cheaper cost of borrowing for the firm. Another approach is to assume that debt carries a convenience yield for households, which can be modeled by allowing firm debt to enter the household's utility function. Thanks to the utility flow, households are willing to hold the debt even if the interest rate is lower than the inter-temporal discount rate $1/\beta - 1$.

As we show below, Assumption 19.4.1 implies that firms prefer debt over equity: they raise debt and pay out dividends. Thus, the tax benefit breaks the indifference property of the financial decisions of firms and introduces a pecking order in the source of funds. This also implies that firms would like to issue an infinite amount of debt. To prevent excessive borrowing, then, it is customary to impose a borrowing limit. We follow that approach here.

Assumption 19.4.2 *Given the price of capital, p_t, the firm's borrowing is subject to the following constraint*

$$\frac{b_{t+1}}{\tilde{R}_t} \leq \xi p_t k_{t+1}.$$

The constraint implies that debt cannot be larger than a fraction ξ of the value of capital. Capital acts as collateral and effectively limits the amount of debt that firms can issue since only a fraction ξ of its value can be funded with debt. The remaining fraction, $1-\xi$, must be funded with equity, that is, funds that are owned directly by the firm and, indirectly, by households. Collateralized credit is a very common practice in bank lending. Although we do not attempt to justify the borrowing constraint with a deeper theory, it can be derived from limited enforcement assumptions.[6]

[6] For example, if in the event of default the lender is only able to confiscate a fraction ξ of capital, the recovered value will be $\xi p_t k_{t+1}$. Knowing this, the borrower could default whenever the debt exceeds this value. But because of this the lender will not be willing to lend an amount greater than this value.

In general, the price of capital p_t plays an important role since its fluctuations affect the firm's access to credit. In our model, however, p_t is always one since it is always possible to convert one unit of capital into a unit of consumption. In many studies, however, it is assumed that the conversion of capital to consumption goods or vice versa, is not one-to-one. This is the case if there are adjustment costs: if we want to increase the stock of capital by one unit, we need to give up more than one unit of consumption. Furthermore, as we add more and more capital, the needed units of consumption increase. This is the idea of convex capital adjustment costs. In this case the price of capital p_t is given by the marginal cost, which increases with investment. For example, convex adjustment costs are present in the financial accelerator model of Bernanke et al. (1999). When the adjustment cost is prohibitively high, the stock of capital is constant and we have, effectively, an economy with a non-reproducible asset similar to land. This is the assumption made in Kiyotaki and Moore (1997). In both papers, the endogenous change in the price of capital plays an important role because it affects the agent's ability to borrow.

To keep the analysis of this chapter tractable, we do not introduce capital adjustment costs, which would make the price of capital move endogenously. Instead, we simplify by assuming that the price of capital, p_t, is exogenous but depends on aggregate shocks. Of course, this is a strong simplification, since prices are endogenous objects determined in equilibrium. The simplification, however, allows us to derive intuitive analytical results.

The lower cost of debt, combined with the binding borrowing constraint (Assumptions 19.4.1 and 19.4.2), introduces a wedge that distorts real allocations. However, this particular distortion plays only a limited role in shaping the economy's response to aggregate shocks. To understand why, suppose that the economy experiences an increase in productivity or TFP. Also, suppose that the higher TFP is expected to persist for several periods in the future, so that firms have an incentive to increase investment. Assumption 19.4.1 implies that the most attractive source of funding for investment is debt since it is cheaper than equity. However, because of the borrowing constraint, only part of the increase in investment can be funded with debt. The other part needs to be funded with equity. In practice, equity financing involves paying lower dividends. Dividends could be negative, which in the model is equivalent to new equity issuance (new shares sold to households). Since there is no cost to adjusting dividends, the firm can always obtain the extra funds by issuing equity. Thus, the borrowing constraint imposes a limit only on one source of funds (debt) but it does not restrict the other source of funding (equity). By having unlimited access to equity, even if more costly, the firm is able to fund any desired level of investment. The firm retains sufficient financial flexibility, so its response to shocks is only marginally affected by the borrowing limit. For financial frictions to play a more significant role, we must also restrict access to alternative sources of funding; in particular, to equity financing.

One way to limit the use of equity financing is to assume that the firm cannot pay negative dividends. The firm can re-invest its current profits by paying zero dividends, but it cannot issue new equity (e.g., sell new shares). This approach has been used in many models in the macro and corporate-finance literatures. Here, we take a more general approach and make the following assumption.

Assumption 19.4.3 *Firms have a target level of dividends, denoted by ϕ, and incur a cost* *when actual payments deviate from this target*

$$\kappa(d_t - \phi)^2.$$

The convexity assumption can be justified on several grounds. When dividends become negative, the firm is effectively raising external equity by issuing new shares. Hansen and Torregrosa (1992) and Altinkilic and Hansen (2000) show that underwriting fees display increasing marginal costs with the size of the offering. Adjustment costs can also be interpreted as capturing managers' preferences for dividend smoothing, as documented by Lintner (1956) and confirmed by subsequent studies. Such behavior may reflect agency conflicts that induce managers to stabilize dividend payments—an aspect that we do not model explicitly. In practice, firms implement dividend policies that are smoother than their corporate earnings, a feature we capture in reduced form by assuming a quadratic cost function penalizing deviations from the target dividend level ϕ.

19.4.1 Optimality conditions for firms

We now have all the elements to characterize the optimization problems of firms and households. The firm's optimization problem can be written recursively as

$$\Omega(S;k,b) = \max_{d,\ell,k',b'} \left\{ d + \mathbb{E}m'\Omega(S';k',b') \right\} \tag{19.3}$$

subject to

$$F(z,k,\ell) - w\ell + \frac{b'}{\tilde{R}} = b + p\Big[k' - (1-\delta)k\Big] + \varphi(d)$$

and

$$\xi p k' \geq \frac{b'}{\tilde{R}},$$

where we have used the function $\varphi(d) = d + \kappa(d-\phi)^2$ to denote the dividend payment to households (shareholders) plus the cost of deviating from the targeted dividend. To pay d to shareholders, the firm uses $\varphi(d)$ units of resources. The function $\Omega(S;k,b)$ is the expected discounted value of dividends. The firm's value depends on the aggregate state, S, and on the individual states given by the beginning-of-period stock of capital, k, and the debt inherited from the previous period, b. The value of the firm is given by the current dividend, d, plus the expected next period value of the firm, $\mathbb{E}m'\Omega(S';k',b')$. The discount factor m' is not affected by the firm's policy. This is because the firm is atomistic and its impact on aggregate variables is negligible. The aggregation of the policies chosen by all firms, however, does affect m'.

The problem is subject to two constraints. The first one is the budget constraint. On the left-hand side are the sources of funds: the firm's operating profits, $F(z,k,\ell) - w\ell$, and the proceeds from new debt issuance, $b'/\tilde{R}$. On the right-hand side are the uses of funds: repayment of outstanding debt, b, capital investment, $p[k'-(1-\delta)k]$, and resources needed to pay dividend payments, $\varphi(d)$. The second constraint to the firm's problem is the borrowing limit. This imposes an upper bound to the capital that can be funded with debt: The left-hand side is the collateral value of capital, while the right-hand side represents the funds raised with new debt.

To characterize the firm's optimal policies, we derive the first-order conditions with respect to dividends, d; labor, ℓ; the new stock of capital, k'; and the new debt, b'. The conditions lead to

$$F_3(z,k,\ell)=w, \tag{19.4}$$

$$\mathbb{E}m'\left(\frac{\varphi'(d)}{\varphi'(d')}\right)\Big[(1-\delta)p'+F_2(z',k',\ell')\Big]=(1-\mu\xi)p, \tag{19.5}$$

and

$$\mathbb{E}m'\left(\frac{\varphi'(d)}{\varphi'(d')}\right)\tilde{R}=1-\mu, \tag{19.6}$$

where $F_i(\cdot,\cdot,\cdot)$ represents the derivative with respect to the ith argument, $\varphi'(\cdot)$ represents the first derivative, λ is the Lagrange multiplier associated with the budget constraint (which was substituted out in this expression) and $\mu\lambda$ the Lagrange multiplier of the borrowing constraint. See Appendix 19.A for details on the derivation.

The optimality condition for labor, equation (19.4), equalizes the marginal product of labor to the wage rate—the same condition as in the standard neoclassical growth model. Therefore, financial frictions do not directly distort firms' optimal labor demand. This is not the case, however, for capital investment. The optimality condition for investment, equation (19.5), equates the effective cost of buying an extra unit of capital (the right-hand side term) to its expected discounted gross return (the term on the left-hand side). If the borrowing constraint is binding, implying that the multiplier μ is positive, the effective cost of an additional unit of capital is lower than its price. The effective cost is lower because the extra unit of capital allows the firm to borrow more since capital can be used as a collateral. Provided that the cost of debt is lower than the cost of equity—Assumption 19.4.1—this is a benefit for the firm as it reduces the effective cost of capital. Notice that the benefit increases with the parameter ξ, since a higher ξ allows each unit of capital to be funded with more (and cheaper) debt. In the literature, the collateral benefit of capital is often referred to as 'collateral premium.' Agents are willing to hold collateralizable capital even if its direct return is lower than other assets because of the collateral benefit.

To gain some intuition about the conditions under which the borrowing constraint binds, we use equation (19.6). This condition tells us that, keeping everything else constant, a lower cost of debt, $\tilde{R}$, is associated with a higher value of the multiplier, μ. A lower cost of debt makes borrowing more attractive and, consequently, increases the value of relaxing the borrowing constraint. In general, the borrowing constraint is more likely to bind when the effective cost of borrowing, $\tilde{R}$, is lower than the inverse of the expected discount factor m'. The reason we use the term 'likely' is because the firm's effective discount factor, $m'\varphi'(d)/\varphi'(d')$, depends also on $\varphi'(d)/\varphi'(d')$. This captures the dividend smoothing motive of the firm, similar to consumption smoothing for households, which is captured by the stochastic discount factor $m'=\beta u_1(C',L')/u_1(C,L)$.

19.4.2 Optimality conditions for households

Households maximize their expected lifetime utility by choosing consumption, c, labor supply, ℓ; the purchase of bonds from firms, b'; and firms' shares, a'. The households' optimization problem can be written recursively as

$$V(S; a, b) = \max_{c,\ell,a',b'} \Big\{ u(c, \ell) + \beta \mathbb{E} V(S'; a', b') \Big\} \tag{19.7}$$

subject to

$$(d + q)a + b + w\ell = c + qa' + \frac{b'}{R} + T.$$

We denote with q the price of one share which pays the dividend d. The household lends b'/R today and gets repaid b' in the next period, with R denoting the gross interest rate. Notice that $R > \tilde{R}$ since $\tilde{R} = R/(1+\tau)$ and τ is positive. The variable T denotes lump-sum taxes paid by the household to cover the interest subsidies to firms. More specifically, this variable is equal to $T = B'/\tilde{R} - B'/R$, that is, the difference between what firms receive by borrowing, $B'/\tilde{R}$, and what households pay, B'/R, must be covered by taxes. We use capital letters to denote aggregate debt, distinguishing it from the individual households' debt choice, b'.

Denoting the Lagrange multiplier associated with the budget constraint by γ, the first-order conditions of problem (19.7) lead to:

$$u_1(c, \ell) w = -u_2(c, \ell), \tag{19.8}$$

$$\frac{1}{R} = \mathbb{E}\left(\frac{\beta u_1(c', \ell')}{u_1(c, \ell)}\right), \tag{19.9}$$

and

$$q = \mathbb{E}\left[\left(\frac{\beta u_1(c', \ell')}{u_1(c, \ell)}\right)(d' + q')\right]. \tag{19.10}$$

The derivation is contained in Appendix 19.A.2. The first equation is the standard optimality condition for the household's supply of labor: it equalizes the utility value of the wage rate earned with one unit of labor (the left-hand side) to the corresponding disutility from working an extra unit of time (the right-hand side). The second condition is the optimality condition for bond holdings, which in equilibrium determines the price of the bond (i.e. the interest rate). Finally, the third condition is the optimality condition for share purchases, which in equilibrium determines the price of an equity share. From the last two conditions we can see that the household discounts next period payments by $\beta u_1(c', \ell')/u_1(c, \ell)$, which is also the discount factor for firms.

19.5 Characterization in a two-period version of the model

To gain intuition, it is helpful to consider a simplified version of the model with only two periods where time runs from date 0 to date 1. We specify the utility and production functions as

$$u(c_t, \ell_t) = \ln(c_t - \ell_t^\nu)$$

and

$$F(z_t, k_t, \ell_t) = z_t k_t^\alpha \ell_t^{1-\alpha}.$$

An additional simplification is to abstract from uncertainty. Therefore, in period 0, agents can predict what will happen in period 1. Because time ends after period 1, we impose the terminal conditions $k_2 = 0$ and $b_2 = 0$. This means that all resources are consumed and debt is fully repaid in period 1. We then solve the model backward: we first solve for the equilibrium allocation in period 1 and then we solve for the allocation in period 0. For the rest of this section we indicate variables in period 0 without a subscript and variables in period 1 with a prime superscript. In equilibrium, $\ell = L$, $k = K$, $b = B$ and $d = D$.

Terminal period, $t = 1$**.** Given the state variables in the terminal period—z', K' and B'—we find the supply of labor by solving condition (19.8) after replacing the wage rate w with the marginal product of labor (condition (19.4)). Using the specified functional forms for utility and production, the condition becomes

$$(1-\alpha)z'\left(\frac{K'}{L'}\right)^\alpha = \nu L'^{\nu-1}.$$

Solving for labor we obtain

$$L' = \left(\frac{1-\alpha}{\nu}\right)^{\frac{1}{\alpha+\nu-1}} (z')^{\frac{1}{\alpha+\nu-1}} (K')^{\frac{\alpha}{\alpha+\nu-1}}.$$

Thus, labor increases in TFP, z', and capital, K', as they both raise the productivity of labor. Plugging L' into the production function we obtain

$$Y' = \left(\frac{1-\alpha}{\nu}\right)^{\frac{1-\alpha}{\alpha+\nu-1}} (z')^{\frac{\nu}{\alpha+\nu-1}} (K')^{\frac{\nu\alpha}{\alpha+\nu-1}},$$

which also increases in z' and K'. In the terminal period all resources are consumed. Therefore, consumption is equal to production, Y', plus non-depreciated capital, $(1-\delta)K'$,

$$C' = (1-\delta)K' + \left(\frac{1-\alpha}{\nu}\right)^{\frac{1-\alpha}{\alpha+\nu-1}} (z')^{\frac{\nu}{\alpha+\nu-1}} (K')^{\frac{\nu\alpha}{\alpha+\nu-1}}.$$

Finally, the argument of the utility function can be expressed as a function of z' and K', that is,

$$C' - L'^\nu = g(z', K').$$

The function $g(z', K')$ is increasing in z' and K', at least in the relevant range, as we can verify by replacing C' and L' with the expressions derived above. This is intuitive because a more productive economy, either because of a higher value of z' or K', will produce more and allows for higher consumption net of the dis-utility from working. Notice that we are making a simplifying assumption: in the terminal period the firm does not incur any cost in the payment of dividends if they deviate from ϕ.

Initial period, $t = 0$**.** The economy starts the initial period 0 with states z, K and B. Firms choose labor demand, L, the new capital stock, K', and the new debt, B'. Households choose labor supply and bond holdings. These decisions are determined by the same first-order conditions derived from the infinite horizon model.

Labor and output are given by the same expressions derived for period 1, but using z and K instead of z' and K'. We focus on the equilibrium in which the borrowing constraint is binding. This would be the case if initial debt B is sufficiently large. Solving for the equilibrium in period 0 entails finding the values of K', B', C, D, R and μ. Having six unknowns, we need six conditions.

The first one is the optimality condition for capital accumulation, equation (19.20). Using the functional forms specified for the two-period model, the condition can be written as

$$(1-\mu\xi)p = \beta\left(\frac{C-L^{\nu}}{g(z',K')}\right)\Big[1-\delta+F_k(z',K',L')\Big]\Big[1+2\kappa\Big(D-\phi\Big)\Big], \tag{19.11}$$

with the marginal product of capital given by

$$F_k(z',K',L') = \alpha\left(\frac{1-\alpha}{\nu}\right)^{\frac{1-\alpha}{\alpha+\nu-1}} z'^{\frac{\nu}{\alpha+\nu-1}} K'^{\frac{\nu\alpha}{\alpha+\nu-1}-1}.$$

Since we assumed that the borrowing constraint is binding and $\mu > 0$, we can use the borrowing constraint as the second condition,

$$\frac{(1+\tau)B'}{R} = \xi p K'. \tag{19.12}$$

The budget constraint for the firm provides the third condition

$$\alpha F(z,K,L) + \frac{(1+\tau)B'}{R} = B + p\Big[K'-(1-\delta)K\Big] + D + \kappa\Big(D-\phi\Big)^2. \tag{19.13}$$

Next we can use the households' budget constraint,

$$D + B + (1-\alpha)F(z,K,L) = C + \frac{B'}{R} + T, \tag{19.14}$$

where taxes T are equal to $\tau B'/R$ and we used the fact that the total shares satisfy $A = A' = 1$ in equilibrium.

The fifth and sixth conditions are given by the first-order conditions for the issuance of new debt by firms and the purchase of the debt by households:

$$\frac{(1-\mu)(1+\tau)}{R} = \beta\left(\frac{C-L^{\nu}}{g(z',K')}\right)\Big[1+2\kappa\Big(D-\phi\Big)\Big] \tag{19.15}$$

and

$$\frac{1}{R} = \beta\left(\frac{C-L^{\nu}}{g(z',K')}\right). \tag{19.16}$$

Equations (19.11) through (19.16) allow us to solve for K', B', C, D, R and μ under the assumption that the borrowing constraint is binding. This is the equilibrium where financial frictions are more relevant. However, if the borrowing constraint is not binding, to solve for the equilibrium we simply replace the borrowing constraint with $\mu = 0$. This would be the case if B is very small or even negative.

In the next subsections we will use the six conditions (19.11)-(19.16) to characterize the responses of the economy to shocks and to show how the responses are affected by financial frictions.

19.5.1 Financial frictions and propagation of shocks

We start considering a positive and fully persistent productivity shock, that is, an increase in z and z'. The implications are summarized in the following property.

Property 19.5.1 *The response of investment to the productivity shock is positive. Current and future output will also increase. However, the responses of investment and output will be smaller if the borrowing constraint is binding.*

The fact that the macroeconomic responses could be smaller when the borrowing limit is binding implies that financial frictions could dampen the macroeconomic response to productivity shocks rather than amplifying it. This may be surprising but it will become clear once we walk through the equilibrium conditions.

A higher productivity in the next period increases the incentive to invest so that firms would like to increase K'. This can be seen in equation (19.11): keeping everything else constant, an increase in z' raises the marginal productivity of capital $F_k(z', K', L')$. To re-establish equality, K' must rise. Of course, other variables will also change, so that the overall change is more complex. But what we describe here captures the main channel shaping the response to a productivity shock. The increase in capital needs to be financed in some way by firms. When the firm chooses a higher value of K', the right-hand side of the budget constraint—equation (19.13)—increases. Thus, something on the left-hand side has to increase or something on the right-hand side has to decrease. The increase in current productivity raises profits, the term $\alpha F(z, K, L)$. This provides some extra funds that can be used to finance investment. Also, a higher value of K' allows the firm to raise more debt as shown in equation (19.12). This increases $(1+\tau)B'/R$ on the left-hand side of the budget constraint. However, provided that $\xi < 1$, which is typically the case, the increase in borrowing and the extra profits will not be sufficient to fund the desired increase in capital. Therefore, in order for the firm to fund the higher investment, it has to reduce dividends, D. Now consider the implications from paying lower dividends. Looking at equation (19.11), a lower value of D reduces the last term and discourages capital investment. The intuition is that reducing dividends has a cost for the firm. If paying lower dividends is necessary to fund investment, the effective cost of investing will be higher and, as a result, the firm invests less than in the absence of this cost.

How should we interpret this in the real world? Due to financial frictions, large investments need to be funded at least in part with equity. But equity is more expensive than other sources of funds, and it becomes more expensive the higher the amount of equity that needs to be raised. This implies that the need to fund investment with equity increases the cost of capital for the firm and discourages investment.

We have shown that, with financial frictions, the response of investment and output could be smaller than in absence of frictions. If that is the case, then financial frictions would dampen the macroeconomic response to shocks rather than amplifying it. The next question is whether there are other mechanisms, besides what we have described here, that could amplify the response of the economy.

19.5.2 The price of capital

An important branch of the macro-finance literature, including the seminal contributions of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), makes the price of capital endogenous so that p also changes in response to the shock. This is important because the ability to borrow depends on the market value of capital through the borrowing constraint,

$$\frac{(1+\tau)B'}{R} \leq \xi p K'.$$

We could think that the price of capital p increases when the economy experiences a boom, especially if persistent, because an economic expansion is typically associated with a higher demand for capital. An increase in the price of capital raises the net worth of firms because the capital they own is now more valuable. Furthermore, since part of the capital is funded with debt, the value of net worth increases proportionally more than the increase in the price of capital. Higher net worth, then, provides firms with more resources to fund the acquisition of capital for the next period. We summarize the impact of a change in the price of capital in the following property.

Property 19.5.2 *If the borrowing constraint is binding and the firm starts with positive debt $B > 0$, a positive price change (higher p) has a positive impact on capital accumulation. This increases next period production directly, through a higher input of capital, and indirectly through higher employment.*

To illustrate this idea, consider the net worth of the firm at the beginning of the period. This is the value of capital minus debt, $pK - B$. Provided that $B > 0$, a 1 percent increase in p generates an increase in net worth larger than 1 percent.

Let's now consider the firm's budget constraint (19.13) and use equation (19.12) to eliminate the new value of debt. We can then re-arrange the budget constraint as

$$K' = \left(\frac{1}{1-\xi}\right)\left((1-\delta)K + \frac{\alpha F(z,K,L) - B - D - \kappa\Big(D-\phi\Big)^2}{p}\right).$$

The term $\alpha F(z,K,L) - B - D - \kappa\Big(D-\phi\Big)^2$ represents the profits net of outstanding liabilities and dividend payments. Typically, this term is negative because the of stock debt is greater than profits (which is a flow). Therefore, an increase in p raises the second term in brackets and results in a higher value of K'. The first term in parentheses on the right-hand side acts as a multiplier, and depends positively on ξ, that is, the fraction of capital that can be used as collateral. Intuitively, a larger ξ allows firms to be more leveraged and, as a result, a change in the price of capital has a larger impact on the firm's net worth. Higher net worth allows the firm to finance more capital.

In summary, an increase in the price of capital generates a proportionally higher increase in the net worth of leveraged firms. The increase in net worth relaxes the borrowing constraint and allows for more investment. This is the central amplification mechanism that operates in Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), as well as in many other models proposed in the macro-finance literature. The basic mechanism illustrated here is also present in the financial accelerator model of Bernanke et al. (1999) and Brunnermeier and Sannikov (2014).

19.5.3 Financial shocks

As described in the previous subsection, the amplification of real shocks could be induced by the impact of the shock on the price of capital. This increases the net worth of borrowers, which in turn relaxes the borrowing constraint. In some cases, however, changes in the tightness of the borrowing constraint are driven by forces that originate directly in the financial sector. This would be captured in our model by a change in the parameter of the borrowing constraint ξ.

There are many mechanisms or channels that could be captured, in reduced form, by a change in ξ. Financial innovations such as securitization could enhance the collateral use of certain assets. This allows borrowers to fund a larger share of investment with debt. Additionally, balance-sheet difficulties experienced by banks could force them to raise credit standards for loan approvals. Figure 19.2 showed that bank credit standards are very cyclical in the data. Based on these considerations, some studies in the macro-finance literature assumed that ξ follows a stochastic process and referred to changes in ξ as 'financial shocks' (Jermann and Quadrini, 2012). The goal of this subsection is to understand the macroeconomic implications of these shocks in the context of the simplified two-period model. We summarize the implications as follows.

Property 19.5.3 *If the borrowing constraint is binding, a negative financial shock (lower ξ) has a negative impact on capital accumulation. This reduces next period production directly, through a lower input of capital, and indirectly through lower employment.*

If the borrowing constraint (19.12) is binding, and keeping other things unchanged, a drop in ξ causes a drop in B'. Looking now at the budget constraint of the firm, equation (19.13), we see that the drop in B' reduces the left-hand side of the budget constraint and, therefore, the right-hand side must also fall. This can be done either by paying fewer dividends D, or by cutting investment, that is, choosing a lower value of K'.

Consider first what happens if the firm reduces dividends. From the first-order condition that characterizes the optimal borrowing of firms, condition (19.15), we can see that the multiplier μ is likely to increase. Intuitively, since the borrowing constraint is tighter when ξ is lower, the value of relaxing the constraint increases. Remember that the Lagrange multiplier μ represents the value of relaxing the borrowing constraint.

We now turn our attention to the equilibrium condition for investment. Lower dividend payments, D, reduce the right-hand side of equation (19.11). The intuition is that when a firm must finance investment by reducing dividends—a costly adjustment—the marginal product of capital has to increase. This can be accomplished by reducing K'. We also notice, however, that the left-hand side of condition (19.11) also changes as μ increases (as we observed above) and ξ decreases (which is the shock). Keeping ξ unchanged, the decline in the left-hand side induced by the lower value of μ is smaller than the decline in the right-hand side induced by a lower D. This is because μ is multiplied by ξ, which is typically smaller than 1. At the same time, the value of ξ is now smaller which, for a given μ, increases the left-hand side of condition (19.11). The intuition here is that capital is not only an input of production but it is also a collateral. Using capital as collateral is valuable because it allows the firm to borrow at a lower cost than through equity. Therefore, when the borrowing constraint becomes tighter (higher μ), the fraction of capital ξ that can be used as collateral becomes more valuable, reducing the effective cost of capital. This, however, is counterbalanced by the fact that ξ is now lower. To summarize, a reduction in the value of ξ has two effects. It reduces the right-hand side of condition (19.11) and it has an ambiguous effect on the left-hand side of (19.11). However, even if the left-hand side drops in value, the drop is smaller than in the right-hand side. Then, in order to re-establish the equality between the left and right sides of equation (19.11), the marginal product of capital $F_k(z', K', L')$ must rise, which requires a lower value of K'. This shows that a decline in ξ (negative financial shock) has a negative impact on investment.

As far as current employment is concerned, we observe that ξ does not affect current employment (see above derivation of L). However, it impacts indirectly next period employment by affecting capital accumulation. From the expression that defines L' derived above, we can see that labor increases in K' and, therefore, lower investment adversely affects next period's employment.

The effects described here require that the borrowing constraint is binding or at least occasionally binding. This brings us back to the relevance of Assumptions 19.4.1 and 19.4.2: financial frictions create an investment wedge (that is, it distorts investment) and a negative financial shock makes that wedge bigger.

19.5.4 Asymmetric responses

An interesting feature of the model with financial shocks is the potential asymmetry with which the economy responds to financial shocks of different sign. Suppose that we start from an equilibrium in which the borrowing constraint (19.12) is not binding. This could arise, for example, if the initial capital K is high or the initial debt B is low. Starting from this equilibrium, the economy is hit by a positive credit shock, that is, an increase in ξ. Since firms were not constrained before the shock, an increase in borrowing capacity does not alter their behavior: if they were not borrowing up to the limit before—that is, their optimal borrowing was below the constraint—there is no reason for them to change that level of borrowing when the limit expands. Hence, the shock has no consequences for the economy.

Property 19.5.4 *If the borrowing constraint is not initially binding, then an increase in ξ has no effect on investment, output, or employment (current and future).*

Now consider a negative financial shock, that is, a decline in the value of ξ. Provided that the decline in ξ is sufficiently large, the firm will no longer be able to borrow the same amount. In other words, condition (19.12) will no longer be satisfied if the firm chooses the same policies in absence of the change in ξ. The change in policies then will have the effects described in the previous subsection, leading to a contraction in investment and a macroeconomic contraction in the next period. This is more likely to arise if the initial capital, K, is low and the initial debt, B, is high.

Property 19.5.5 *If the borrowing constraint is not initially binding, then a sufficiently large decrease in ξ reduces investment, future output, and future employment.*

The asymmetric feature of financial shocks is one of the reasons they have been used to understand the dynamics of financial booms and busts: while financial booms tend to be gradual and long lasting in the data, financial busts tend to be more sudden with sizable macroeconomic implications. The asymmetry depends on the financial structure of the firm when the shock hits. As argued above, the asymmetry is a direct consequence of binding borrowing constraints which in turn depend on K and B. But what determines the initial capital and debt? What values are more relevant?

Unfortunately, the two-period model is silent about the determination of the initial states, since K and B are exogenous. In this regard, the infinite horizon model provides additional insights, as both K and B are endogenously determined. In particular, through simulations, we can derive the invariant distributions of capital and debt. This informs us about the likelihood that certain states emerge in equilibrium and allows us to determine how common binding constraints are. It also informs us about the history that could lead the economy to states where binding constraints become likely. We will show this numerically in the last section of this chapter.

19.5.5 Financial frictions and the labor wedge

So far, we have illustrated a particular mechanism through which financial frictions could impact the macro-economy. That mechanism operates through the optimality condition for investment. Financial frictions do not distort directly the optimality condition for labor. In fact, condition (19.4) is exactly the same as in the standard neoclassical model. Labor is affected only indirectly through capital accumulation. An implication is that, while the amplification mechanism created by financial frictions could be important for the dynamics of investment, it could be somewhat negligible for the dynamics of labor. Considering that labor contributes significantly to business cycle dynamics, this would make financial frictions less relevant for understanding the business cycle. To see this, recall the growth accounting framework introduced in Sections 2.1.4 and 14.4.3, which defines the Solow residual as $\ln z_t = \ln Y_t - \alpha \ln K_t - (1-\alpha)\ln L_t$, and is used to construct measures of TFP. Setting $\alpha = 0.36$, and using annual data from 1950 to 2019 for the U.S. economy, we find that cyclical movements in labor contribute about 60 percent to the standard deviation of GDP. The contribution of capital, instead, is less than 10 percent. The remaining contribution can be attributed to cyclical fluctuations in TFP.

There are two reasons why movements in the stock of capital have a small contribution to output fluctuations. First, even though capital expenditures are very volatile, they are only a small fraction of the stock of capital. This implies that K does not move much over the business cycle, even if investment does. Second, movements in K are multiplied by the share α, which is smaller than 0.5. Movements in labor, instead, are multiplied by $1-\alpha$ which is bigger than 0.5. Therefore, in order for financial frictions to play a significant role in aggregate output fluctuations, they must impact labor directly. One way to do that is with the introduction of 'working capital.'

Suppose that the payment of wages needs to be made before firms receive the revenues from their sales. This is a feature of the production cycle where costs and revenues are not perfectly synchronized. Because of this, firms need to carry liquid funds from the previous period or borrow additional funds to pay for the wages. Carrying liquid funds from the previous period or borrowing in the current period have similar implications. However, the latter is more convenient in terms of notation. We thus assume that firms borrow at the beginning of the period to make advanced payments of wages. This type of borrowing is in addition to the intertemporal debt already introduced in the model. Since the debt raised to pay wages is repaid within the period, there are no interest payments.

With the working capital extension we have two types of borrowing: the intra-period debt, which is equal to the wage bill, wl, and the inter-temporal debt b'/R. The total debt, sum of the two types of borrowing, is subject to the same limit as before, that is,

$$w\ell + \frac{b'}{R} \leq \xi p k'. \tag{19.17}$$

The problem solved by the firm is still given by (19.3) but with the borrowing constraint taking the form specified in (19.17). The optimality conditions for capital and intertemporal borrowing are (19.5) and (19.6), which are the same as before. The first-order condition for labor, however, changes to

$$F_3(z,k,\ell) = (1+\mu)w. \tag{19.18}$$

This shows that the marginal product of labor is equalized to the wage rate only if the multiplier μ is zero, that is, if the borrowing constraint is not binding. However, if the borrowing constraint is binding, labor will be distorted, and the distortion increases in the tightness of the borrowing constraint, captured by the multiplier μ. This introduces an interesting mechanism through which financial frictions can have significant implications for aggregate economic activity: tighter financial conditions are reflected in higher values of the multiplier μ which, for a given wage rate, reduce labor demand.

Productivity and financial shocks influence the macroeconomy in similar ways to those discussed earlier, but now also affect labor demand directly. We show this again with the two period model where, for simplicity, we assume that wages are paid in advance only in period 0. Thus, the working capital constraint does not apply in the terminal period 1. This also implies that the equilibrium in period 1, given the states z', K' and B', is equivalent to the one characterized earlier. We state the main property as follows.

Property 19.5.6 *Given productivity z and capital K, employment declines with the tightness of the borrowing constraint, which is captured by the multiplier μ.*

In period 0, equilibrium labor is found by solving condition (19.19), adjusted for the presence of working capital. Using the specific functional forms for utility and production, the condition is

$$(1-\alpha)zK^{\alpha}L^{-\alpha} = (1+\mu)\nu L^{\nu-1}.$$

Solving for labor we obtain

$$L = \left[\frac{1-\alpha}{(1+\mu)\nu}\right]^{\frac{1}{\alpha+\nu-1}} z^{\frac{1}{\alpha+\nu-1}} K^{\frac{\alpha}{\alpha+\nu-1}}.$$

Labor continues to increase in productivity, z, and capital, K. In addition, it now depends negatively on the multiplier μ. This is because, as the borrowing constraint becomes tighter and the value of μ increases, firms demand less labor and, in equilibrium, employment is lower. We could repeat the same analysis conducted earlier with the two-period model and obtain similar results. The most important addition is that a decline in the price of capital p and/or a decline in ξ generates a macroeconomic contraction also in period 0 since it affects directly the demand for labor. This is in addition the to impact on investment and on next period production as described earlier.

We conclude this subsection by pointing out that the need for working capital does not derive only from the advanced payment of wages. It could also derive from the advance payment of intermediate inputs. The production function typically used in the standard neoclassical growth model abstracts from the need of intermediate inputs. Instead, it focuses on the reduced form where final goods are produced only with the inputs of capital and labor. In reality, though, any type of production requires intermediate goods that are produced by other firms and may need financing. With a more complex production process, tighter financial conditions may cause a macroeconomic contraction by distorting the intermediate-good inputs.

19.6 General model and the neoclassical growth model

We now move to analyze the infinite horizon model. The equilibrium can be characterized by combining the first-order conditions of households and firms together with market clearing conditions. Replacing the wage rate in equation (19.8), and using equation (19.4), we obtain

$$F_3(z, K, L) = -\frac{u_2(C, L)}{u_1(C, L)}. \tag{19.19}$$

The marginal product of labor is always equalized to the marginal rate of substitution between consumption and leisure, which is the same condition for the standard neoclassical growth model.

We can rewrite the optimality condition for the accumulation of capital, equation (19.5), as

$$(1-\mu\xi)p = \mathbb{E}\left(\beta\frac{u_1(C', L')}{u_1(C, L)}\right)\left(\frac{\varphi'(D)}{\varphi'(D')}\right)\Big[(1-\delta)p' + F_2(z', K', L')\Big]. \tag{19.20}$$

Compared to the standard NGM, we observe two important differences. First, the effective cost of capital (the left-hand side) is not the price p but $(1-\mu\xi)p$ reflecting the collateral value of capital discussed earlier. The second difference with the standard NGM is that the return from capital received next period (e.g., the marginal product plus the resale value) is discounted at a different rate. The effective discount factor used by firms includes the term $\varphi'(D)/\varphi'(D')$, which is absent in the standard model. Obviously, this term disappears if we relax Assumption 19.4.3, eliminating any cost associated with dividend payments. If we also relax Assumption 19.4.2, then $\mu = 0$ and the equilibrium condition for capital would be exactly identical to the standard NGM.[7]

We now turn to the equilibrium in the bond market (the debt issued by firms and purchased by households). Using $\tilde{R} = R/(1+\tau)$, equation (19.6) can be rewritten as

$$\frac{(1-\mu)(1+\tau)}{R} = \mathbb{E}\left(\beta \frac{u_1(C', L')}{u_1(C, L)}\right)\left(\frac{\varphi'(D)}{\varphi'(D')}\right), \tag{19.21}$$

while the first-order condition for the households' bonds' choice, equation (19.6), is

$$\frac{1}{R} = \mathbb{E}\left(\beta \frac{u_1(C', L')}{u_1(C, L)}\right). \tag{19.22}$$

If we relax Assumption 19.4.3 by imposing $\kappa = 0$ (no cost in deviating from the dividend target), conditions (19.21) and (19.22) imply $(1-\mu)(1+\tau) = 1$. Therefore, a sufficient condition to have a binding borrowing constraint (so that $\mu > 0$) is that $\tau > 0$. This is Assumption 19.4.1. However, if $\kappa > 0$, which implies $\varphi(D) \neq D$, the borrowing constraint may not be always binding. For example, if in expectation $\varphi'(D)/\varphi'(D')$ is sufficiently larger than 1, then condition (19.21) cannot be satisfied with $\mu > 0$. This could arise if the firm would like to pay more dividends now relatively to those paid in the next period. Another contingency in which this may arise is when, in expectation, the term $u_1(C', L')/u_1(C, L)$ is higher. For example, when future households' consumption is lower than current consumption. The property that the borrowing constraint is only occasionally binding has been extensively studied in the financial crises literature, where crises can be interpreted as episodes in which the constraint becomes binding. We will come back to this point in the next section when we conduct a numerical analysis.

To summarize, we have shown that Assumptions 19.4.1, 19.4.2, 19.4.3 introduce wedges in the equilibrium condition for capital accumulation that distort allocations. In the absence of these three assumptions we would revert back to the standard neoclassical model as stated formally in the following proposition.

Proposition 19.1 *If $\tau = 0$ and $\kappa = 0$, the equilibrium debt chosen by firms is indeterminate and the real allocation is the same as in the standard neoclassical growth model.*

Proof. The proof is obtained by comparing the equilibrium first-order conditions after setting $\tau = 0$ and $\kappa = 0$. We have already shown that with $\tau = 0$ and $\kappa = 0$, the multiplier associated with the borrowing constraint is $\mu = 0$. Conditions (19.20) and (19.21) then become

$$p = \beta \mathbb{E}\left(\frac{u_1(C', L')}{u_1(C, L)}\right)\left[(1-\delta)p' + F_2(z', K', L')\right]$$

and

$$\frac{1}{R} = \beta \mathbb{E}\left(\frac{u_1(C', L')}{u_1(C, L)}\right).$$

[7] For consistency, however, we cannot relax Assumption 19.4.2 while keeping Assumption 19.4.1. Otherwise the firm would borrow an infinite amount of debt. Thus, the relaxation of Assumption 19.4.2 is done jointly with the relaxation of Assumption 19.4.1.

These two conditions, together with condition (19.19), characterize the equilibrium. Since they are exactly the same conditions as in the neoclassical model, the equilibrium allocation is the same. ■

If $\tau = 0$, the cost of debt for the firm, $\tilde{R}$, is equal to the interest rate, R. If $\kappa = 0$, there is no cost in deviating from the targeted dividend ϕ. This is equivalent to eliminating Assumptions 19.4.1 and 19.4.3. In equilibrium, then, firms are indifferent between funding investment with debt or equity. This is true even if there is a borrowing limit as assumed in Proposition 19.4.2. This is another example of the Modigliani-Miller principle we discussed in Section 19.3.1.

More intuition for Modigliani-Miller principle

Consider a change in financial structure where the firm raises Δ units of funds with debt and uses the funds to pay dividends. By doing so the firm increases its liabilities and reduces its equity. In the next period, then, the firm reduces the payment of dividends to repay the debt inclusive of interests, ΔR. By doing so, the firm increases dividend now by Δ and reduces the next period dividends by ΔR. In present value, the next period dividends are worth $\mathbb{E}m'\Delta R$. Therefore, the gain from changing the financial structure is $\Delta(1 - \mathbb{E}m'R)$. When $\tau = \kappa = 0$, the first-order condition (19.6) simplifies to $\mathbb{E}m'R = 1$. This implies that the gain from changing the sources of funding is zero.

The proposition also says that the equilibrium allocation remains the same as in the standard neoclassical model, even if investment decisions are made by firms instead of households. Intuitively, this is because firms use the same discount factor as households and therefore make the same decisions that households would.

19.7 Numerical analysis

We now use the infinite-horizon model with working capital to show some of the properties numerically. While a two-period model is useful to gain intuition, it necessarily takes the initial states as given. As we observed in the analysis of the two-period model, whether the equilibrium features binding or non-binding borrowing constraints depends on the initial states. Thus, the two-period model does not offer much guidance on whether the borrowing constraint binds or, more generally, on the severity of financial frictions.

19.7.1 Calibration

The households' utility takes the form

$$u(c, \ell) = c^{\nu}(1 - \ell)^{1-\nu}.$$

We calibrate the model at a quarterly frequency and impose that the price of capital is constant and equal to 1 ($p = 1$). As observed above, many components of the model are similar to the RBC model. Therefore, for the parameters that are common to the two models, we use the values typically used in the RBC literature. These parameters include the discount factor $\beta = 0.983$, the weight of consumption in the utility function $\nu = 0.412$, the share of capital in the production function $\alpha = 0.36$, and the process for productivity z. It is customary to assume that TFP follows a first-order Markov process (see Chapter 10, Section 10.4). We do the same here but we assume that the shock takes only three values (a finite-state Markov chain). We calibrate these values together with the transition probability matrix to match the correlation and standard deviation of the empirical series of Solow residuals, as done in the RBC literature. The mean value of z is not important as it acts as a normalization factor.

At this point, we have the values for the standard parameters and we are left with the non-standard parameters. They include the parameters that determine the firm's benefit of debt over equity, τ; the cost of dividend deviations, κ; and those determining the stochastic process for the variable ξ (e.g., the financial shock). These parameters do not appear in the RBC model and, therefore their calibration requires a more detailed explanation.

We start with τ, the tax benefit of debt for the firm. Since corporations can deduct interest payments when determining corporate tax liabilities, the effective cost of debt is reduced by the corporate tax rate. Assuming a marginal corporate tax rate of 28.1%, we set $\tau = 0.281$. Let us try now to understand which variables are more likely to be affected by the remaining parameters, starting with the stochastic process for ξ. As for productivity, we assume that ξ follows a three-state Markov process. The mean value of ξ has a direct effect on the average debt chosen by the firm. Therefore, one empirical moment we can use as a calibration target is the average leverage observed in the data (the ratio of debt, B, over physical capital, K). A second moment we can use is the persistence of debt in the data (autocorrelation) since in the model the persistence of debt will be related to the persistence in ξ. Finally, we can use the volatility of debt in the data (standard deviation) as the third targeted moment: a more volatile ξ will generate a more volatile debt in the model. Thus, the empirical measures of mean, autocorrelation and standard deviation of debt will be the three moments we can use to calibrate the parameters that determine the stochastic process for ξ. The last parameter we need to calibrate is κ. The higher the value of κ, the lower the volatility of dividends. Based on this, we can use the empirical volatility of dividends as a calibration target. Since dividends in the model capture not only dividends but more generally payout to shareholders, we can use the volatility of equity payout, defined as dividends plus share repurchases minus new equity issuance. The full set of parameters with their calibrated values are reported in Table 19.1.

19.7.2 Numerical solution of the model

The model is solved numerically using a global method. We first discretize the state space for K and B/K on a two-dimensional grid and iterate on the optimality conditions using the projection method. The discretization of B/K—rather than B—is convenient because, given the borrowing limit $B/\tilde{R} \leq \xi K$, the admissible values of B change with K while the admissible values of B/K are independent of K. The detailed computational steps are described in the Online Appendix. The Online Appendix also provides the codes to replicate the results shown here.

Table 19.1: Parameter values.

Parameters	Description	Values
β	Discount factor	0.983
ν	Utility parameter	0.412
α	Capital share in production	0.360
τ	Benefit of debt	0.281
κ	Dividends' cost	0.500
p	Price of capital	1.000
z	Productivity and transition probabilities	$\begin{pmatrix} 0.183 \\ 0.185 \\ 0.187 \end{pmatrix}, \begin{bmatrix} 0.900 & 0.075 & 0.025 \\ 0.050 & 0.900 & 0.050 \\ 0.025 & 0.075 & 0.900 \end{bmatrix}$
ξ	Debt limit and transition probabilities	$\begin{pmatrix} 0.450 \\ 0.500 \\ 0.550 \end{pmatrix}, \begin{bmatrix} 0.900 & 0.075 & 0.025 \\ 0.050 & 0.900 & 0.050 \\ 0.025 & 0.075 & 0.900 \end{bmatrix}$

19.7.3 Simulation exercise

The numerical analysis is based on the following thought experiment. Suppose that the economy experiences a long sequence of productivity and financial shocks $z = 0.185$ and $\xi = 0.500$. These are the middle values of the three possible realizations and correspond to the mean of the two shocks. Even if the long sequence of shocks have the same values, agents do not anticipate them. Thus, in every period, they predict the next period realizations using the conditional probabilities. After the long sequence of realizations, the economy converges to an equilibrium that remains constant until different realizations of the two shocks are drawn. Starting from this equilibrium, we assume that the realization of productivity switches from $z = 0.185$ to $z = 0.187$—the highest possible realization of productivity—and stays there for several periods. The draw of the financial shock, instead, remains at $\xi = 0.500$ over the whole simulation. The engineered switch in z is meant to capture a productivity boom and the goal of the exercise is to explore how the various endogenous variables respond to the productivity boom.

To compare the productivity boom to a productivity decline, we repeat the simulation just described but with productivity that switches to $z = 0.183$—the lowest value—and stays there for some periods. Also in this case the draw of the financial shock remains $\xi = 0.500$ over the whole simulation. The simulation captures a productivity downturn. The dynamics of debt and output around the two productivity switches are plotted in Figure 19.3.

The switch in productivity arises at quarter zero. We can see that the productivity expansion and contraction lead to the opposite but symmetric dynamics of debt and output. Panel (C) plots the Lagrange multiplier μ, which takes a positive value when the borrowing limit is binding. As can be seen, μ is always positive meaning that the borrowing constraint is always binding over the whole simulation.

Are the responses of financial variables consistent with the empirical facts outlined by Figures 19.1 and 19.2? The responses of debt and output are both positive, implying positive co-movement as in the data. However, we observe that the response of debt is relatively small compared to the response of output while the data show the opposite pattern. What about interest rate spreads? In the model we do not have interest rate spreads because there is not default. However, to the extent that interest rate spreads capture the tightness of

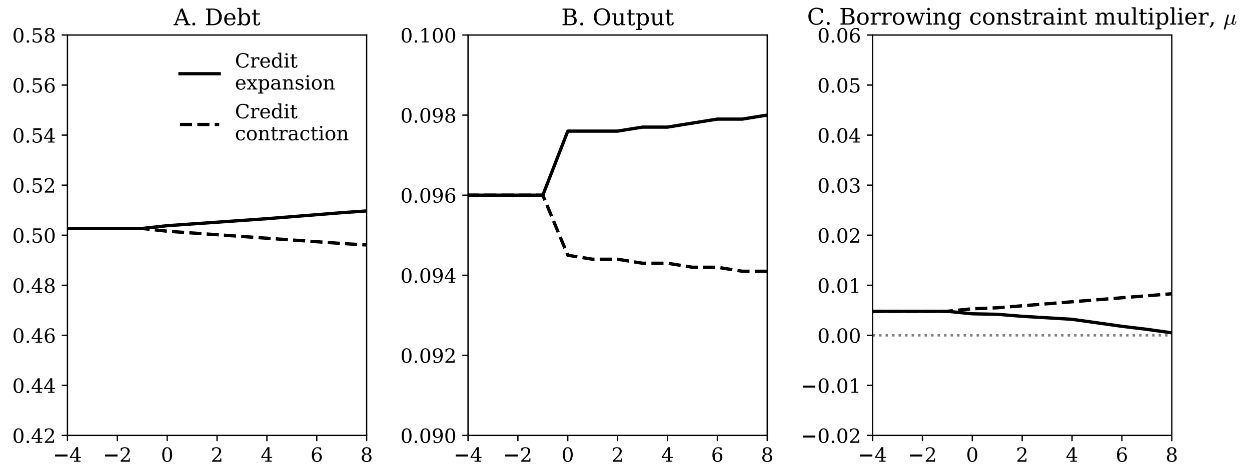

Figure 19.3: Response of debt, output, and borrowing constraint multiplier to a persistent productivity shock.

financial constraints, we can compare the dynamics of the multiplier μ to the spreads in the data. As shown in Figure 19.2, spreads are highly counter-cyclical, which is also the case for the responses of μ to productivity shocks. A similar observation can be made regarding loan standards which can be interpreted as indicators of financial tightness. In the model, financial tightness is captured by the multiplier μ. Both loan standards in the data and μ in the model are counter-cyclical. Our simulation assumes that the price of capital p is fixed. However, if the price of capital increases in a productivity boom, it could relax the borrowing constraint and lead to a much lower value of μ. Conceptually this is possible but for this to be important quantitatively, the movements in p must be sizable. It turns out that this is challenging to achieve in models that maintain the basic structure of the RBC framework.

We now turn our attention to financial shocks. We conduct a similar quantitative exercise but with changes in the financial variable ξ. We assume that the economy experiences a draw of a long sequence of productivity and financial shocks $z = 0.185$ and $\xi = 0.500$. At some point, however, the realization of the financial variable increases from $\xi = 0.500$ to $\xi = 0.550$, and stays at the higher level for several periods. The draw of productivity, instead, remains at its mean value $\xi = 0.185$. This corresponds to a credit boom. We then repeat the simulation but this time assuming that the financial variable switches to the lower value $\xi = 0.450$, while productivity remains constant throughout the simulation. The dynamics of debt, output and multiplier are shown in Figure 19.4.

Even though the switch in the financial shock is symmetric (the increase in ξ in the credit expansion is equal to the decrease in ξ in the credit contraction), the responses of debt and output are asymmetric. A positive financial shock leads to a gradual increase in debt and to a relatively persistent but mild increase in output. On the contrary, a negative financial shock leads to a sharp decline in debt and to a very large contraction in output. The contraction, however, is not very persistent. Panel (C) shows that the borrowing constraint becomes non-binding after the credit expansion, as the multiplier becomes zero.

What is the intuition for the dynamics shown in Figure 19.4? Once ξ reaches the highest value $\xi = 0.550$, there is always the possibility of reversal which would force firms to re-adjust

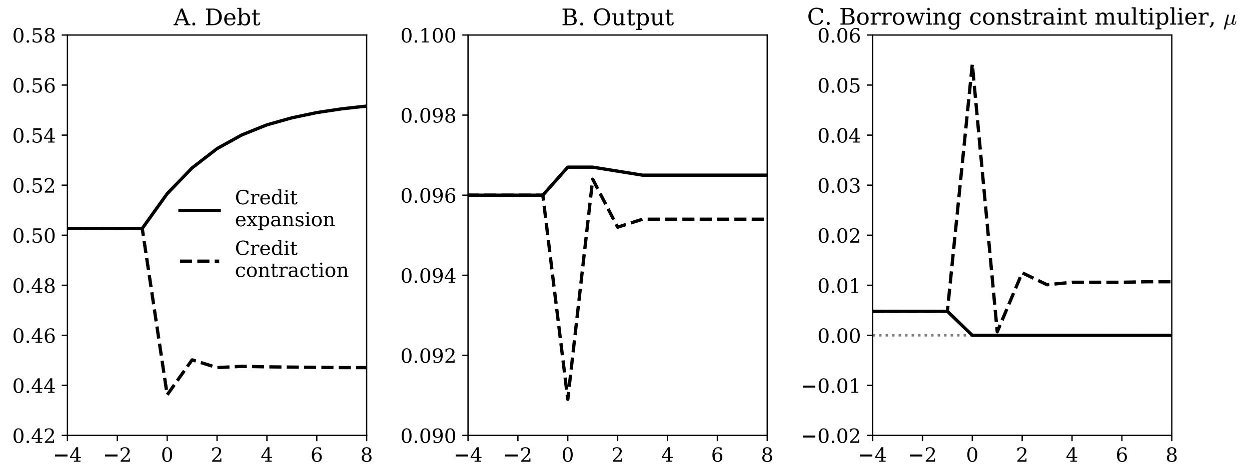

Figure 19.4: Response of debt, output, and borrowing constraint multiplier to a persistent financial shock.

their debt if they borrow up to the limit. But this will be very costly because it requires the firm to either reduce dividends (which is costly because of $\varphi(\cdot)$) or to reduce labor and investment. This creates a precautionary motive that induces firms to borrow less than the limit, even if debt is cheaper than equity. When the financial shock is negative, the firm is forced to cut borrowing. In order to do so, the firm needs to cut labor and investment (so that less financing is needed) or pay lower dividends (so that the reduction in debt can be compensated by an increase in equity). Both choices are inefficient and the firm chooses the optimal combination that minimizes the inefficiencies.

The qualitative properties of the dynamics induced by financial shocks shown in Figure 19.4 are consistent with the stylized facts outlined in the empirical literature. Reinhart and Rogoff (2009) and Schularick and Taylor (2012), for example, have shown that many episodes of credit booms are not associated with much faster growth in real economic activity. However, when the credit boom experiences a sudden stop, the reversal is often characterized by a sharp macroeconomic contraction. The dynamics displayed by the model are also consistent with the empirical dynamics shown in Figures 19.1 and 19.2. In addition to having pro-cyclical debt, we also have that financial tightness (interest rates spreads and loan standards in the data, and multiplier μ in the model) is counter-cyclical.