

Chapter 24

Sovereign debt and default risk

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24.1 Introduction

The bulk of this textbook assumes agents have full commitment to repay their debts. In this chapter, we remove that full-commitment assumption. An immediate implication is that the debt sustainability calculations from Chapter 15—which were based on constant risk-free rates—no longer hold. When repayment is uncertain, interest rates become endogenous, rising with the perceived probability of default. Sustainability assessments must therefore account not only for debt levels and fiscal policy, but also for the incentives and constraints that shape repayment decisions.

Historically, sovereign defaults have been most closely associated with emerging markets, most notably the wave of debt crises across Latin America in the 1980s, when a combination of high global interest rates, collapsing commodity prices, and excessive borrowing triggered widespread defaults. However, sovereign risk is by no means confined to developing economies. The European debt crisis of 2011–2012 revealed similar vulnerabilities in advanced economies, as Greece, Portugal, Spain, and Ireland faced surging borrowing costs and questions about their solvency within the Euro area. These episodes highlight that the underlying mechanisms we study—how rising debt and default risk interact to constrain borrowing and raise interest rates—operate across economies of heterogeneous size and income levels, from emerging markets to advanced economies.

A further motivation for studying sovereign default comes from a distinctive set of business cycle patterns, particularly evident in emerging markets. As documented in Chapter 23, these economies exhibit greater output volatility and, more strikingly, consumption that is more volatile than output—contrary to the standard consumption-smoothing logic. This excess volatility is closely tied to their external borrowing behavior, which tends to be strongly procyclical: countries borrow heavily in booms and repay during recessions. One leading explanation is that credit access shrinks in downturns.

These patterns point directly to the mechanisms at the heart of our analysis. To understand them, we need a framework in which sovereign risk shapes borrowing opportunities and, in turn, macroeconomic volatility. This chapter develops such a model. We begin by documenting key features of business cycles and sovereign debt markets across countries. We then introduce a canonical sovereign-default model in which a benevolent government bor-

rows abroad but can choose to default. The model captures the central tension between the desire to smooth consumption over time and the temptation to renege on debt obligations. Unlike the frictionless models in Chapters 23 and 15, we explicitly incorporate endogenous default risk.

24.2 Empirical patterns

We begin by documenting key features of business cycles and sovereign debt markets across countries: the excess volatility of consumption, the cyclical behavior of sovereign spreads, the frequency and consequences of default episodes, and the patterns of “debt intolerance.”

24.2.1 Excess volatility of consumption

[Aguiar and Gopinath \(2007\)](#) and [Neumeyer and Perri \(2005\)](#) notice key differences in the dynamics of aggregate variables in advanced vs. emerging-market economies. They show that while aggregate consumption is less volatile than income in small open advanced economies, it is more volatile than income in emerging economies. The extra consumption volatility is at odds with the standard consumption-smoothing motive, which prescribes that households save a fraction of the extra income earned during economic booms to buffer the consumption drop during recessions. As shown in Table 23.1 in Chapter 23, this pattern remains evident in more recent data: the consumption-to-output volatility ratio is higher in emerging markets than in advanced economies, and the overall volatility of output is also significantly larger. Although the differences are somewhat less pronounced than those documented by [Aguiar and Gopinath \(2007\)](#), they persist and continue to motivate the theoretical frameworks developed in this chapter.

The extra consumption volatility in emerging economies is accounted for by their external borrowing behavior. In an open economy, agents can dissave by liquidating capital or by accumulating net external liabilities. As shown by Table 23.1, there are no significant differences in investment behavior between advanced and emerging economies, implying that most of the extra consumption volatility in emerging economies is accounted for by their external borrowing behavior. This is reflected in a more countercyclical trade balance. A typical pattern in several emerging economies is to borrow from the rest of the world to finance trade balance deficits during booms, and partially repay their external liabilities by running trade balance surpluses during recessions. Again, this is at odds with standard consumption smoothing.

What accounts for the apparently puzzling borrowing behavior of emerging economies? [Alvarez-Parra, Brandao-Marques, and Toledo \(2013\)](#), [García-Cicco, Pancrazi, and Uribe \(2010\)](#), and [Neumeyer and Perri \(2005\)](#) argue that borrowing opportunities play an important role. They present different versions of the small open economy model expanded with various shocks and the following (simplified) aggregate budget constraint:

$$C_{t+1} + K_{t+1} = Y_t + K_t(1 - d) + q(Y_t, \eta_t, B_{t+1})B_{t+1} - B_t. \quad (24.1)$$

Here, C denotes aggregate consumption, Y the level of output, K the stock of capital with depreciation rate d , B denotes the net external liabilities, q denotes the price at which

the economy can issue debt, and η denotes a shock to the price q .¹ [Alvarez-Parra et al. \(2013\)](#), [García-Cicco et al. \(2010\)](#), and [Neumeyer and Perri \(2005\)](#) find that a price q that increases with income and decreases with debt plays a key role in accounting for the strong countercyclical trade balance, and thus, procyclical external borrowing.

24.2.2 Sovereign defaults

Both the academic and policy literature on sovereign defaults typically use the definition of a default event proposed by credit-rating agencies. This definition identifies as a default event every episode in which the sovereign makes a debt restructuring offer with terms that are less favorable to creditors than the original debt terms. Thus, default events include both legal defaults in which the sovereign breaches the original debt contract, and “pre-emptive” debt restructurings (renegotiations of debt terms before a payment has been missed). [Asonuma and Trebesch \(2016\)](#) find that 38% of debt restructurings between 1978 and 2010 were pre-emptive.

The most frequently used measure of creditors’ losses after a debt restructuring is the present-value “haircut,” which is defined as one minus the ratio of present value of new to old debt obligations

$$Haircut = 1 - \frac{\sum_{t=1}^{\infty} \frac{x_t^{Post}}{(1+i)^t}}{\sum_{t=1}^{\infty} \frac{x_t^{Pre}}{(1+i)^t}}, \quad (24.2)$$

where x_t^{Post} denotes the post-restructuring debt payment obligations in period t , x_t^{Pre} denotes the pre-restructuring debt payment obligations in period t , and i denotes the bond yield prevailing immediately after the restructuring (see, e.g., [Sturzenegger and Zettelmeyer, 2005](#)). [Cruces and Trebesch \(2013\)](#) calculate that the average present-value haircut in sovereign debt restructurings is 37%.

Sovereign defaults have been a relatively frequent event in emerging economies. The updated dataset of [Asonuma and Trebesch \(2016\)](#) and [Asonuma, Niepelt, and Ranciere \(2017\)](#) records 201 sovereign defaults to private creditors between 1970 and 2020. [Figure 24.1](#) shows that, at the end of 2022, emerging economies’ sovereign debt in default represented 2% of their GDP.

Sovereign default facts

[Aguiar and Amador \(2014\)](#) summarize the key characteristics of sovereign defaults. First, they tend to occur in bad times. [Tomz and Wright \(2007\)](#) find that more than 60% of sovereign defaults occurred in years when the GDP was below trend. This is consistent with the aforementioned countercyclicity of the borrowing cost in emerging economies. Secondly, sovereign defaults are followed by years of negotiations between the sovereign and its creditors. Thirdly, defaults end when sovereigns settle their debt

¹If the economy saves (it chooses $B_{t+1} < 0$) by buying foreign bonds that pay the global risk-free interest rate r , the bond price is $q = 1/(1+r)$.

in default with a debt restructuring (in which defaulted debt instruments are exchanged for new debt instruments). [Asonuma and Trebesch \(2016\)](#) and [Asonuma et al. \(2017\)](#) report an average of 3.2 years between the onset of default and its settlement with creditors, with a significant heterogeneity in default durations and haircuts. For example, preemptive restructurings have an average negotiation time of 12 months, and post-default restructurings have an average negotiation time of 60 months.

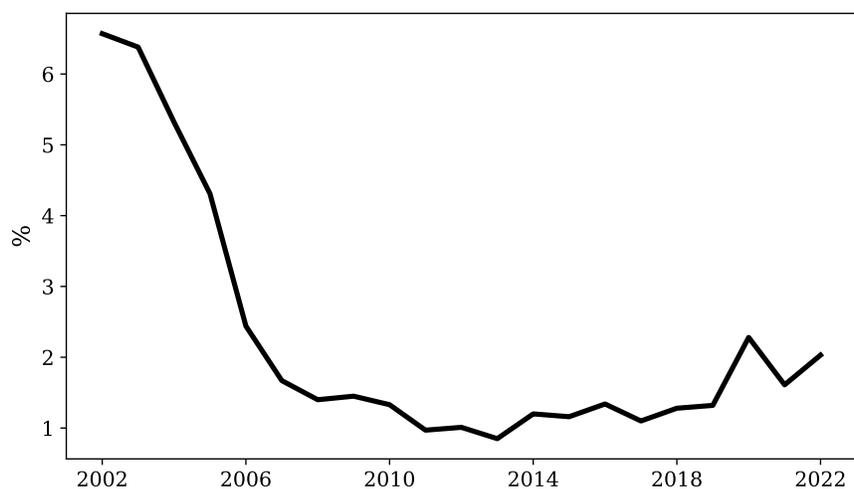


Figure 24.1: Emerging economies' public debt in default.

Notes: The series is expressed as a percentage of the combined GDP of emerging economies. Source: 2023 Bank of Canada Sovereign Default Database.

24.2.3 Sovereign spreads

As a consequence of default risk, sovereign bond yields command a premium over safer bonds.² The Emerging Markets Bond Index (EMBI) computed by J.P. Morgan is the most commonly used reference for bond yields for sovereign debt in emerging economies. The index measures the sovereign yield spread, i.e., the extra yield of U.S. dollar-denominated bonds over similar bonds issued by the U.S. government.

Table 24.1 shows that sovereigns in emerging economies pay a significant and volatile premium when they borrow from international markets. Also, the fourth column shows that the premium increases when the GDP is below trend. The high, volatile, and countercyclical sovereign spreads here are consistent with the findings in [Alvarez-Parra et al. \(2013\)](#), [García-Cicco et al. \(2010\)](#), and [Neumeyer and Perri \(2005\)](#). The last section of this chapter presents a quantitative sovereign default model in which the spread is endogenous and displays the same behavior as in the data. We show that this feature plays an important role in accounting for the distinctive business cycle fluctuations of emerging economies.

²The bond yield at time t represents the hypothetical return an investor would make if it holds the bond until the maturity date and the bond is not defaulted on.

Table 24.1: Sovereign spread yield

	E(Spread)	σ (Spread)	ρ (Spread, GDP)
Argentina	732	377	-0.5
Brazil	514	380	-0.2
Ecuador	1021	647	-0.6
Korea	162	116	-0.7
Malaysia	174	115	-0.3
Mexico	318	221	-0.4
Peru	307	199	-0.1
Philippines	283	172	-0.3
Slovak Republic	57	36	-0.6
South Africa	242	111	-0.3
Thailand	155	112	-0.5
Turkey	385	199	-0.5
Average Emerging	362	224	-0.4

Note: The sovereign spread is expressed in basis points. The spread series is quarterly and spans the period Q1 1994 - Q4 2019 whenever data is available. We removed periods in which the sovereign is in default. The series for aggregate GDP was logged and filtered using the Hodrick-Prescott filter with a smoothing parameter of 1600.

Table 24.2 illustrates the potential determinants of the sovereign spread. It presents the results of a fixed-effects panel regression that estimates

$$\log(\text{Spread})_{it} = \alpha + \beta X_{it} + \delta_i + \eta_{it}, \quad i = 1, \dots, N, \quad t = 1, \dots, T,$$

where i denotes the country index, t denotes the year, X_{it} is a vector of control variables for country-specific and global macroeconomic factors, δ_i are country fixed effects; and η_{it} represents disturbances that are independent across countries and time. The table shows that the spread (i) increases with the total government debt, (ii) decreases with GDP growth, (iii) decreases with foreign reserves owned by the sovereign, and (iv) increases with the VIX index (reflecting global financial conditions).³ While we cannot extract causal relationships from Table 24.2, the findings suggest a strong link between the sovereign spread, sovereign borrowing, domestic shocks to economic activity, sovereign asset accumulation, and global financial shocks.

Costs of sovereign risk. On top of the adverse effects of sovereign risk on economic stability and business cycles discussed above, following the Eurozone crisis, a recent branch of the literature has focused on the adverse effects of sovereign risk on investment and growth. For example, [Arellano, Bocola, and Bai \(2024\)](#) find that a 100 basis points increase of the sovereign spread leads to a 64 basis points increase in firms' borrowing cost. They also calculate that in 2012, if the sovereign spread had not increased, real GDP in Italy would have fallen 3.2 percent instead of 6.4 percent.

³The results in the table are similar to the ones reported in [Jaramillo and Tejada \(2011\)](#), [Akitoby and Stratmann \(2008\)](#), and other studies.

Table 24.2: Panel regressions

	Coeff.	SE
Public debt to GDP	0.020	(0.004)
Real GDP growth	-0.042	(0.006)
Reserves to GDP	-0.033	(0.017)
Net gov borrowing to GDP	0.020	(0.014)
VIX	0.034	(0.003)
Observations	523	
R-squared	0.77	
Number of countries	33	

Note: We use annual data from a sample of 33 emerging market countries spanning from 1994 to 2018. Robust standard errors are in parentheses.

The costs of sovereign risk were also apparent during COVID-19, when countries with higher risk suffered a stronger deterioration of their borrowing opportunities and thus had more limited resources to mitigate the adverse effects of the pandemic. Figure 24.2 shows that the spread increase after COVID-19 was typically larger for countries that were already facing a higher sovereign spread (reflecting higher risk) before the pandemic. In particular, countries with a pre-COVID spread below 300 basis points only suffer a spread increase below 100 basis points.

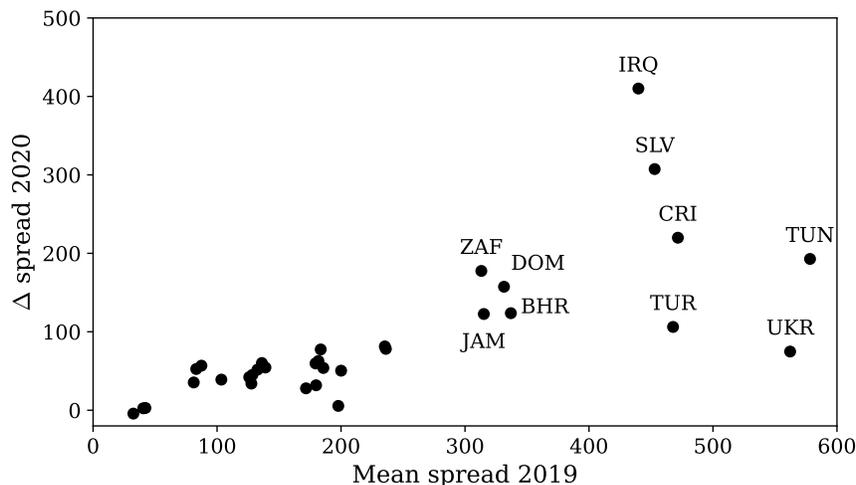


Figure 24.2: EMBI Spread increase after COVID-19.

Notes: The (average daily) spread and spread change (2020 average minus 2019 average) are expressed in basis points. We removed countries with a spread higher than 1000 basis points in 2019 (Argentina, Lebanon, and Venezuela). Source: Bloomberg.

24.2.4 Debt intolerance

The inability of sovereigns to commit to repaying their debt constrains their borrowing capacity. This feature has been emphasized in the literature on debt intolerance. Debt intolerance can be understood as the mapping from public debt to sovereign default risk. Debt intolerance varies both across countries and over time (Reinhart, Reinhart, and Rogoff, 2015; Reinhart, Rogoff, and Savastano, 2003). Figure 24.3 illustrates variations in the mapping between public debt ratios and sovereign default risk (measured using credit default swap spreads). First, the figure shows that there is significant heterogeneity in the debt-spread mapping. Economies like Thailand or Uruguay can feature public debt ratios above 50% while incurring modest default risk, while Turkey incurs a substantial default risk at lower debt ratios. Second, some economies accumulated significant debt without increasing their default risk (e.g., China and Thailand), while others experienced an increase in default risk after debt accumulation (e.g., Brazil). Finally, the mapping from debt and spread significantly shifts over time. For instance, default risk in Portugal, Greece, and Latvia declined significantly between 2010 and 2023, without substantial debt reductions (note also that before 2008, markets were pricing a nearly zero default risk for most Eurozone economies).

We will show how assuming shocks to global risk premia and/or aggregate income, sovereign default models can account for the volatility of default risk observed in the data for one country. However, those shocks alone are insufficient in accounting for the cross-country heterogeneity in the debt-spread mapping and some of the time variation in debt levels within countries. This suggests the presence of structural heterogeneity in default incentives across countries and time.

The difficulty for governments in emerging economies of issuing debt in their own currency has been associated with the debt intolerance problem in these economies. This contrasts with the landscape in advanced economies, which typically issue debt in their own currencies. Eichengreen and Hausmann (1999) refer to the issuance of debt denominated in foreign currency as the “original sin.” Issuing debt in foreign currency is problematic in part because the exchange rate tends to depreciate in bad times, increasing the foreign-currency debt burden (in comparison with the government’s income, which is mostly determined in local currency; Hausmann, 2003).

The left panel of Figure 24.4 shows that the share of sovereign debt issued in foreign currency is positively correlated with the sovereign spread. The difficulty emerging economies have had in issuing debt in their own currency has been linked to pitfalls in their ability to conduct monetary policy (Du, Pflueger, and Schreger, 2020; Engel and Park, 2022; Ottonello and Perez, 2019). Some countries are increasingly overcoming this problem: Du and Schreger (2016) show how sovereigns in emerging economies are borrowing more in their local currency. The right panel of Figure 24.4 shows that many emerging economies have managed to issue less foreign currency sovereign debt.

24.2.5 Remedies

The most notable difference between sovereign debt and household or corporate debt is that the former lacks a well-specified bankruptcy protocol. One avenue that sovereigns in emerging economies have followed to enhance their repayment credibility is to issue debt in

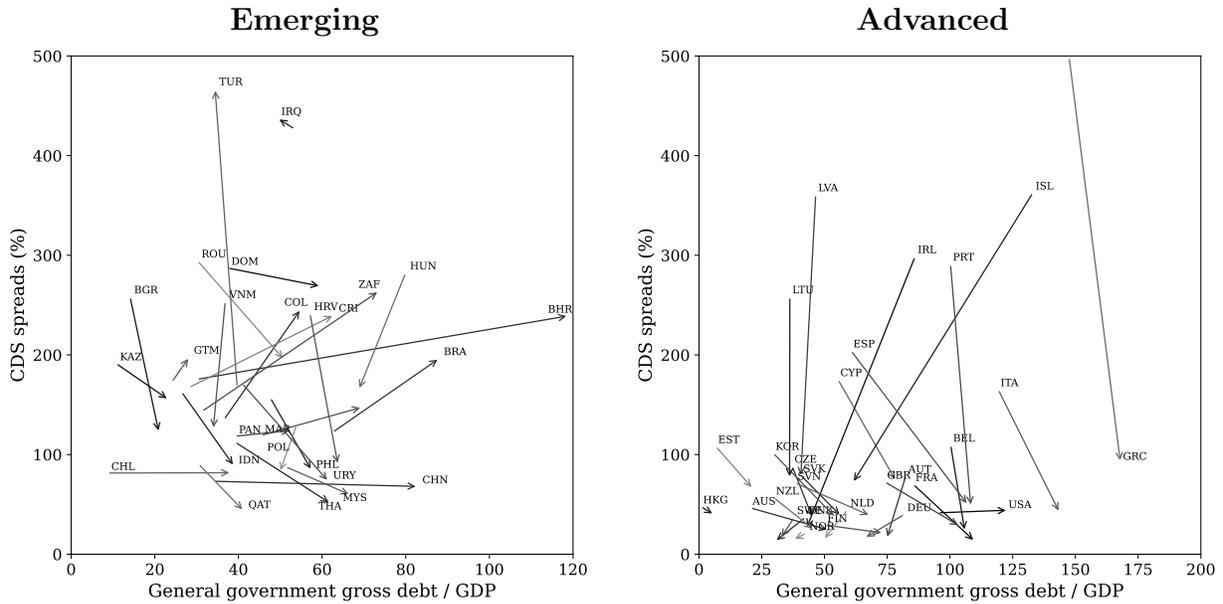


Figure 24.3: Credit default swaps (CDS) spread and public debt ratios for emerging and advanced economies.

Notes: The origin point of each arrow illustrates the debt-spread combination in 2010, and its end point the debt-spread combination in 2023. We use the CDS spread because it enables us to expand the sample of countries to advanced economies.

Sources: Bloomberg and IMF-WEO database.

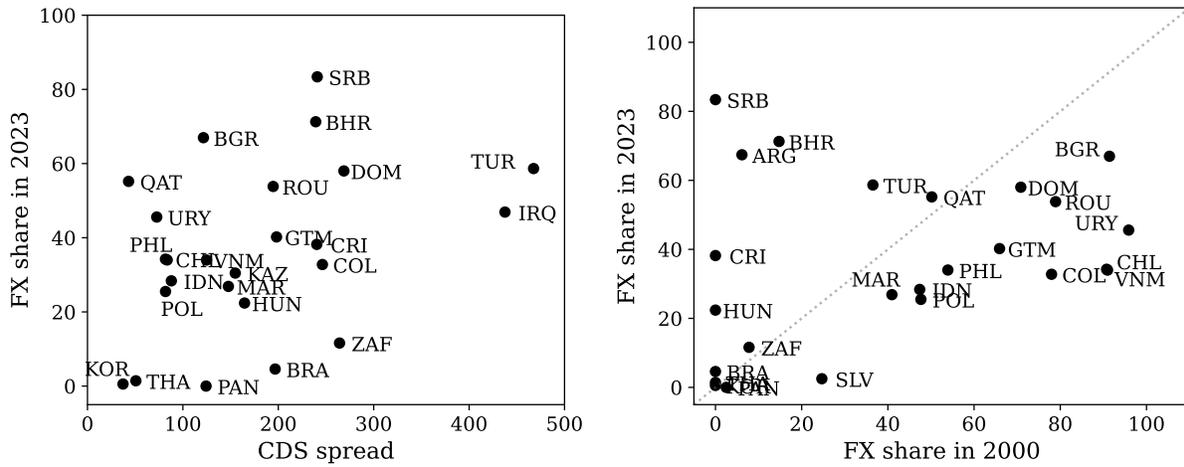


Figure 24.4: Spreads and foreign currency sovereign debt.

Notes: The left panel shows a positive association between the CDS spread (in basis points) and the share of foreign currency sovereign debt in emerging economies. The right panel illustrates how the share of foreign currency sovereign debt in emerging economies changed from 2000 to 2023. Source: WEO database and Bloomberg.

foreign financial centers, and thus under the jurisdictions of foreign courts.⁴ However, in

⁴Sovereigns also increase their cost of defaulting by introducing debt covenants that include acceleration and cross-default clauses. Acceleration clauses allow creditors to call all future payments in a bond in case

contrast to the ability of courts to force payments by private agents, it is very difficult for foreign courts to enforce payments by sovereigns. Thus, while posting collateral is a strategy often followed by households and firms to obtain loans at a lower interest rate, with a few notable exceptions, sovereigns have a limited ability to credibly pledge collateral. This is because the principle of sovereign immunity limits creditors’ right to confiscate sovereign assets.

Fiscal rules as remedies to sovereign risk. Figure 24.5 shows that an increasing number of countries are adopting fiscal rules, in part to limit “excessive” borrowing, sovereign risk, and costly sovereign default episodes.⁵ The figure also shows that the bulk of countries adopting fiscal rules are limiting the debt level (limits to the fiscal budget balance or the fiscal expense also constrain the debt level). [Aguiar, Amador, and Fourakis \(2020\)](#) and [Hatchondo, Martinez, and Roch \(2022\)](#) use a sovereign default model to quantify the benefits of debt limits.

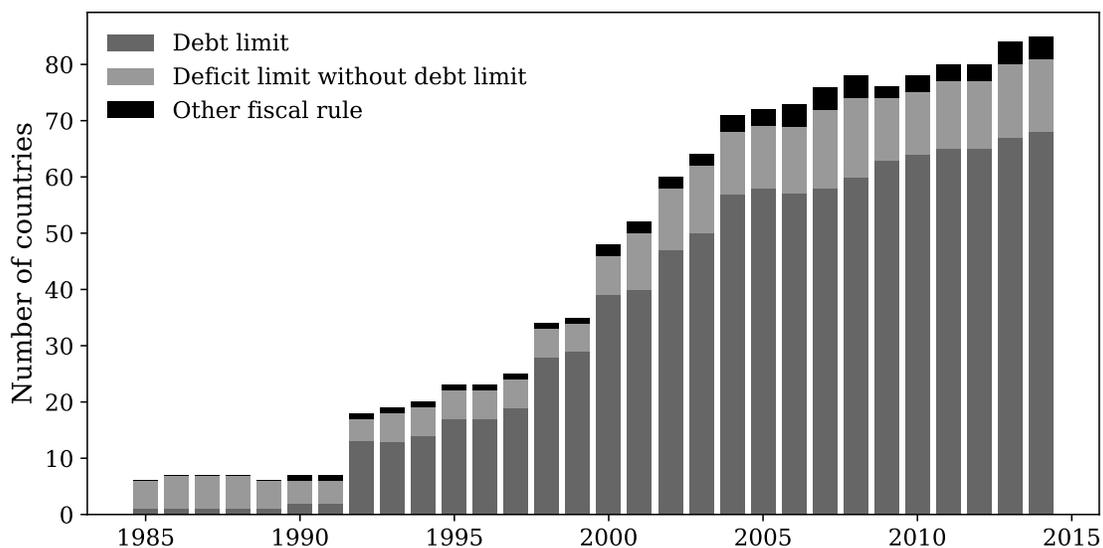


Figure 24.5: Number of countries with fiscal rules.

Source: IMF Fiscal Rules dataset.

However, the variation in debt intolerance across countries and time described above presents challenges for the use of debt levels as anchors in fiscal rules. For example, the variation of debt intolerance across countries makes it challenging to introduce the same debt limit to a group of heterogeneous countries, as with the Maastricht treaty for Eurozone members. The common limit is likely to excessively restrain public borrowing for some of the countries in the group with low debt intolerance (that could otherwise expand debt without significant increases in default risk). At the same time, the common limit may be

the government defaults on one payment of that bond. Cross-default clauses state that a default in any government debt obligation constitutes a default in the contract containing the cross-default clause.

⁵Fiscal rules in Figure 24.5 impose at least one and often more than one numerical target. These targets may limit the level of debt, the budget balance, revenues, and expenditures.

ineffective for economies with high debt intolerance (that could display significant sovereign risk for lower debt levels). Likewise, a time-invariant fiscal rule can become inadequate for economies that present time-varying debt intolerance.

The limitations of debt levels as fiscal anchors were underscored in recent debates about reforming the European Union fiscal framework. [Blanchard, Leandro, and Zettelmeyer \(2020\)](#) propose to replace debt-limit fiscal rules with fiscal standards (i.e., qualitative prescriptions that leave room for judgment) guided by probabilistic analysis that reflects the level of sovereign risk (as done by the IMF for their sovereign risk assessment for market-access countries; [IMF, 2022](#)). Similarly, [Hatchondo et al. \(2022\)](#) propose to use the sovereign spread (reflecting sovereign risk) as the anchor for fiscal rules. [Furman and Summers \(2020\)](#) suggest focusing on the government's interest payments to account for fluctuations in the (risk-free) interest rate.

Low-income countries

One key characteristic of low-income countries is the predominant role of official debt, often in the form of concessional loans (e.g., with an interest rate below the market rate or with grace periods). While official debt is sometimes restructured, the majority of official debt restructurings do not feature face-value write-offs ([Cheng, Diaz-Cassou, and Erce, 2016](#); [IMF, 2023](#)), with some well-known exceptions such as the 1996 Heavily Indebted Poor Countries Initiative. For example, the Debt Service Suspension Initiative for the official bilateral debt of poorer countries implemented during COVID-19 ([Lang, Mihalyi, and Presbitero, 2023](#)) did not include face-value write-offs. Thus, in contrast with the sovereign spread for emerging economies discussed above, the interest rate governments pay for official debt typically does not reflect default risk. The preponderance of official debt can have a significant effect on sovereign risk, which is the focus of the sovereign default literature as well as this chapter. In fact, the ability of governments to obtain debt in concessional terms is the key criterion of the International Monetary Fund for deciding on the framework used for assessing the risk of debt distress in a country ([IMF, 2013](#)). [Horn, Reinhart, and Trebesch \(2021\)](#) discuss the increasing importance of China as a lender. This includes the “Belt and Road” initiative to finance and build infrastructure.

The rest of the chapter presents first a stylized two-period version of the canonical equilibrium default model used to integrate sovereign risk in the analysis of aggregate fluctuations. We then show how the infinite-horizon version of the model (presented in the appendix to this chapter) can account quantitatively for the distinctive features of business cycles in economies with sovereign default risk. In the default model, both the interest rate and the sensitivity of the interest rate to borrowing increase in bad times, generating the counter-cyclical trade balance and a volatility of consumption higher than the volatility of income that is characteristic of economies facing significant default risk..

24.3 A stylized two-period default model

As [Eaton and Gersovitz \(1981\)](#), we focus on a small-open economy model with strategic defaults. Following [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#), an extensive literature uses the infinite-horizon version of this model for the quantitative analysis of aggregate fluctuations. We first present a stylized two-period model that is amenable to an analytical characterization of how default risk constrains the sovereign's choices. We then proceed to presenting the infinite-horizon version of the model used for quantitative analysis.

The small-open economy is populated by a continuum of identical households (who do not make any choices) and a benevolent government that maximizes households' welfare. That is, the domestic economy is modeled as a single agent that we call the government. The economy lasts for two periods, $t = 1, 2$. The economy is endowed with Y_2 goods in period 2.⁶ The endowment realization Y_2 is stochastic. Let F and f denote the c.d.f. and density functions of Y_2 , with $f(Y_2) > 0$ for all $Y_2 \geq 0$. We assume the economy is endowed with $Y_1 = 0$ goods in period 1, and the government enters period 1 with zero debt. This ensures the government needs to borrow in period 1.

The government's objective is to maximize the social welfare function

$$u(C_1) + \beta \mathbb{E}u(C_2),$$

where u denotes the consumers' utility function, C_t denotes consumption in period t , $\beta \in (0, 1)$ denotes households' discount factor, and \mathbb{E} denotes the expectation operator. The utility function satisfies $u' > 0$, $u'' < 0$, and the Inada condition.

In period 1, the government can borrow to finance consumption. A bond issued in period 1 promises to pay one unit of the single good in period 2. Bonds are issued to foreign, risk-neutral investors that operate in a competitive market. To simplify notation, we assume these investors discount future payments at a rate of 1, i.e., they ask for a zero interest rate compensation when purchasing bonds.

The government cannot commit to repaying its debt in period 2. If the government defaults, it does not pay its debt but loses a fraction $\phi > 0$ of Y_2 . For sovereign debt to materialize in equilibrium, it must be more costly for a sovereign to default than to pay back its debt for at least some states of the economy. Without a cost of defaulting, the government would always default and, anticipating this, investors would never lend. At the same time, for sovereign defaults to occur in equilibrium, there must be some states for which it is more costly for the government to pay back its debt than to default. Most of the quantitative default literature presents a simple formulation of the cost of defaulting calibrated to match properties of the sovereign spread in the data. Measuring the effect of default on income is difficult because of a reverse causality problem: while defaults could have a negative effect on income, low income (and the expectation of low income in the future) may push the government to default. In contrast, sovereign spreads are easily measured, and the dynamics of sovereign spreads in the simulations of the quantitative model are determined by the assumptions on the default cost.⁷

⁶Each household is endowed with $y_2 = Y_2$ goods.

⁷The literature discusses as possible costs of sovereign defaults financial penalties in the form of higher borrowing costs and/or exclusion from debt markets for defaulting countries ([Cruces and Trebesch, 2013](#)),

As is standard in the literature, we use the Markov-Perfect equilibrium concept. This is, we assume that in period 1, the government cannot commit to its period-2 default decision. Thus, one may interpret this environment as a game in which the government making decisions in period 1 is a player who takes as given the default strategy of its future self, who will decide in period 2.

Equilibrium borrowing and default

To characterize the equilibrium in this model, we solve it backwards.

Period 2

Let B_2 denote the number of bonds issued by the government in period 1 and that mature in period 2. Let D_2 denote the default decision in period 2, where $D_2 = 1$ (0) indicates that the government defaults (repays). Since there is no borrowing in period 2, consumption in period 2 satisfies

$$C_2 = \begin{cases} Y_2 - B_2 & \text{if } D_2 = 0, \text{ and} \\ Y_2 - \phi Y_2 & \text{if } D_2 = 1. \end{cases} \quad (24.3)$$

We assume that the government repays when it is indifferent between repaying and default. The government decides whether to default in period 2 maximizing utility $u(C_2)$. Therefore, the government picks the highest possible value for C_2 , implying that it defaults if and only if the cost of defaulting (ϕY_2) is less than the cost of paying its debt (B_2). Let \hat{D}_2 and \hat{C}_2 denote the equilibrium default and consumption decisions in period 2. It follows that

$$\hat{D}_2(B_2, Y_2) = \begin{cases} 1 & \text{if } Y_2 < B_2/\phi, \text{ and} \\ 0 & \text{otherwise.} \end{cases} \quad (24.4)$$

and

$$\hat{C}_2(B_2, Y_2) = Y_2 - \hat{D}_2(B_2, Y_2)\phi Y_2 - [1 - \hat{D}_2(B_2, Y_2)]B_2.$$

Period 1

In period 1, investors bid to purchase bonds. Since we assumed investors do not discount the future, they bid a price q_1 that coincides with what they expect to recover from each bond. Since each sovereign bond promises to pay one unit of the good in period 2, what lenders

direct sanctions and trade costs (Asonuma, Chamon, and Sasahara, 2016), reputational spillovers (Cole and Kehoe, 1998 and Amador and Phelan, 2021), and domestic financial and political costs (Broner, Martin, and Ventura, 2010 or Azzimonti and Quadrini, 2024). As presented in the Appendix, the quantitative default model often assumes that the defaulting country is excluded from debt markets for a stochastic number of periods. This assumption does not have a significant effect on the properties of the model. Bulow and Rogoff (1989) show that assuming financial autarchy as the only cost of defaulting would not be enough to support debt in equilibrium.

expect to recover is this one unit multiplied by the probability of repayment. Formally, the equilibrium bond price satisfies

$$q_1(B_2) = \mathbb{E} \left[1 - \hat{D}_2(B_2) \right] = Pr(Y_2 < B_2/\phi) = 1 - F(B_2/\phi). \quad (24.5)$$

In a rational expectations equilibrium, the repayment probability expected by lenders is determined by the government's equilibrium default decision, specified in equation (24.4). The price q_1 is an equilibrium because: i) if investors offered to buy bonds at a higher price than $q_1(B_2)$, they will earn negative profits in expectation, and ii) if they offered a lower price than q_1 , they will be outbid by other investors.

Equation (24.5) illustrates how, in equilibrium default models, borrowing choices available to the government are restricted by the limited commitment problem. The bond price decreases with the number of bonds issued. The government's inability to commit to repaying in period 2 limits how much it can borrow in period 1.⁸

At the beginning of period 1, the government chooses how much to borrow taking as given its own equilibrium default decision in period 2 (equation 24.4) and the implied bond price schedule (equation 24.5). Thus, the government's problem in period 1 consists of

$$\max_{B_2 \geq 0} \left\{ u(C_1) + \beta \mathbb{E} \left[u(\hat{C}_2(B_2, Y_2)) \right] \right\}$$

subject to

$$C_1 = B_2 q_1(B_2).$$

Equilibrium borrowing \hat{B}_2 satisfies

$$u'(C_1(B_2)) \left[q_1(B_2) + B_2 \frac{dq_1(B_2)}{dB_2} \right] = \beta \mathbb{E} \left[u'(\hat{C}_2(B_2, Y_2)) \left[1 - \hat{D}_2(B_2, Y_2) \right] \right]. \quad (24.6)$$

Equation (24.6) shows how the Euler equation in a model with defaults is different from the one in a model without default. The current marginal benefit of issuing an extra bond depends on the value of increasing consumption in period 1 ($u'(C_1)$), and the marginal revenue from issuing an extra bond ($q_1 + B_2 dq_1/dB_2$). In a default model, this marginal revenue depends on the derivative of the bond price because when the government issues an extra bond, it increases the default probability on all the bonds issued in period 1, not only the default probability on the extra bond issued. The right-hand side represents the expected repayment cost of issuing an extra bond. This cost is determined by the additional resources the government would need to take from households in period 2 to repay B_2 , reducing consumption in repayment states (when $\hat{D}_2 = 0$). Since we assumed the government writes off the entire debt stock when it defaults, consumption in default states does not depend on B_2 .

⁸Note that following the majority of the literature and to be consistent with the most commonly used sovereign debt instruments, we are assuming the government issues non-contingent debt. In our stylized model, allowing the government to issue income-contingent debt would expand its borrowing opportunities and eliminate defaults. [Roch and Roldán \(2023\)](#) discuss limitations of state-contingent debt instruments.

24.4 Simulations using a quantitative default model

This section presents simulation results from a quantitative infinite-horizon version of the default model (in which the government faces income shocks and can choose whether to default in every period) calibrated using data from Mexico as reference. We first describe the fully dynamic environment here and define the equilibrium we will study (using dynamic programming). This model is a straightforward extension of the two-period model above, but solving it requires numerical methods and we explain different methods in Appendix 24.A, along with a description of the details of the calibration.

24.4.1 The environment

Preferences. The sovereign's preferences are given by

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u(C_j).$$

The utility function is strictly increasing and concave.

Technology. We assume that the economy receives a stochastic endowment stream of a single tradable and perishable good. The economy's endowment is denoted by $Y \in \mathbb{R}_{++}$. The endowment process follows:

$$\log(Y_t) = (1 - \rho) \mu + \rho \log(Y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

Debt Structure. The sovereign issues long-term bonds. Since the relevant state space for the sovereign consists of the future stream of debt payments, long-term bonds need to be modeled in a way that maintains tractability. We assume that the sovereign issues perpetuities that promise a deterministic stream of coupons that decreases at an exogenous constant rate δ (Hatchondo and Martinez, 2009; Arellano and Ramanarayanan, 2012). In particular, a bond issued in period t promises to pay $\delta(1 - \delta)^{j-1}$ units of the tradable good in period $t + j$, for all $j \geq 1$. Hence, debt dynamics can be represented by the following law of motion:

$$B_{t+1} = (1 - \delta)B_t + I_t, \tag{24.7}$$

where B_t is the number of bonds due at the beginning of period t , and I_t is the amount of bonds issued in period t . The advantage of this formulation is that the current debt stock (B_t) is sufficient for predicting the future stream of debt payments (absent future debt issuance). Equation (24.7) is akin to the equation for capital accumulation in the neoclassical growth model. In that model, agents do not need to know when each unit of capital was incorporated into the economy because all capital units depreciate at a constant rate. Equation (24.7) shows that agents in this model do not need to know when each bond was issued because all bond payments decline at the same rate.

Budget constraint. Let q_t denote the price at which the sovereign issues bonds. When the sovereign has access to debt markets, it faces the following budget constraint:

$$C_t + \delta B_t = Y_t + I_t q_t. \quad (24.8)$$

The sovereign finances consumption (C_t) and coupon payments (δB_t) with income (Y_t) and the resources collected from debt issuance ($I_t q_t$).

Defaults. The sovereign cannot commit to repay its debt obligations and can declare a default in any period. When the sovereign defaults, it does so on all current and future debt obligations.⁹ The canonical model assumes that the recovery rate for debt in default (i.e., the fraction of the loan that lenders recover after a default) is zero (this assumption has been relaxed in several studies and is inconsequential for the issues we underscore in this chapter).

The cost of defaulting is that the sovereign is excluded from debt markets for a stochastic number of periods and suffers an income loss $\phi(Y_t)$ in each period in which the economy is excluded from debt markets. As in [Arellano \(2008\)](#) and [Chatterjee and Eyigungor \(2012\)](#), we assume that it is proportionally more costly to default in good times ($\phi(Y)/Y$ increases in Y). They show that this property is important in accounting for spread dynamics.¹⁰

Bond market. Bonds are priced in a competitive market inhabited by a large number of foreign investors. Thus, the bond price q_t is pinned down by a zero-expected profit condition. Investors are risk-neutral and discount future payoffs at the rate r , representing the global risk-free interest rate and thus the investors' opportunity cost of lending.

Timing. The timing of events within each period is as follows. Firstly, the sovereign and investors observe the income realization. Secondly, the sovereign chooses whether to default on its debt. Thirdly, if the sovereign does not default, it pays current debt obligations and rebalances its debt portfolio.

Equilibrium concept. We use the Markov-Perfect equilibrium concept. The sovereign cannot commit to future default and borrowing decisions. The strategy for the sovereign acting in period t is assumed to depend only on payoff relevant variables, which in this environment consist of debt and income in period t (B_t, Y_t).

24.4.2 Recursive formulation

Let D denote the current-period default decision, where $D = 1$ if the sovereign defaults and $D = 0$ if the sovereign repays. Let V denote the sovereign's value function at the beginning of a period, that is, before the default decision is made. Let V^R denote the value function

⁹This is a standard assumption in the literature and is consistent with the observed behavior of defaulting sovereigns and the widespread presence of the acceleration and cross-default clauses.

¹⁰[Mendoza and Yue \(2012\)](#) shows that this property of the cost of defaulting arises endogenously in a setup in which defaults affect the ability of local firms to acquire a foreign intermediate input good.

of a sovereign that has repaid its debt and V^D denote the value function of a sovereign in default. The function V satisfies the following functional equation:

$$V(B, Y) = \max_{D \in \{0,1\}} \{DV^D(Y) + (1 - D)V^R(B, Y)\}. \quad (24.9)$$

If the sovereign defaults, the economy consumes its available resources $Y - \phi(Y)$. With probability ψ , the sovereign may regain access to bond markets in each of the following periods. Let F denote the conditional cumulative distribution function of the next-period endowment Y' . The function V^D satisfies the following functional equation:

$$V^D(Y) = u(Y - \phi(Y)) + \beta \int [\psi V(0, Y') + (1 - \psi) V^D(Y')] F(dY' | Y). \quad (24.10)$$

Note that the sovereign regains access to debt markets with zero debt and thus, the continuation value of that contingency is $V(0, Y')$.

If the sovereign repays, current consumption is determined by equation (24.8). We avoid a potential discontinuity at $B' = 0$ with the constraint $B' \geq 0$. We verify that this constraint is not binding in the simulations.¹¹ For any bond price function q , V^R satisfies the following functional equation:

$$V^R(B, Y) = \max_{B' \geq 0} \left\{ u(C) + \beta \int V(B', Y') F(dY' | Y) \right\}, \quad (24.11)$$

subject to

$$C = Y - \delta B + q(B', Y) [B' - (1 - \delta)B].$$

For investors to break even, the bond price must equal the expected discounted value of debt payments. Namely:

$$q(B', Y) = \frac{1}{1 + r} \int [1 - \hat{D}(B', Y')] [\delta + (1 - \delta)q(\hat{B}(B', Y'), Y')] F(dY' | Y), \quad (24.12)$$

where \hat{D} and \hat{B} denote the default and borrowing rules lenders expect the sovereign to follow in the next period. The default rule \hat{D} is equal to 1 if the sovereign defaults and is equal to 0 otherwise. The function \hat{B} determines the debt stock chosen by the current sovereign. The first term on the right-hand side of equation (24.12) equals the expected value of the next-period coupon payment promised in a bond. The second term in the right-hand side of equation (24.12) equals the expected value of all other future coupon payments, which is summarized by the expected price at which the bond could be sold next period.

24.4.3 Equilibrium definition

A Markov Perfect Equilibrium is characterized by

1. Value functions: V , V^R , and V^D ,

¹¹In the simulations, the sovereign typically does not even buy back debt (i.e., it never chooses $B' < (1 - \delta)b$). In an environment without income shocks, [Aguiar, Amador, Hopenhayn, and Werning \(2019\)](#) show it is never optimal for the sovereign to buy back debt.

2. a default rule \hat{D} and a borrowing rule \hat{B} , and
3. a bond price function q ,

such that:

(a) given \hat{D} and \hat{B} , V , V^R , and V^D satisfy functional equations (24.9), (24.10), and (24.11), when the sovereign can trade bonds at q ;

(b) given \hat{D} and \hat{B} , the bond price function q is given by equation (24.12); and

(c) the default rule \hat{D} and borrowing rule \hat{B} solve the dynamic programming problem defined by equations (24.9) and (24.11) when the sovereign can trade bonds at q .

24.4.4 Results

Table 24.3 illustrates first how the simulations can match the targeted average levels of debt and spreads in the data.¹² Table 24.3 also shows that the default model can account for distinctive features of business cycles in emerging economies: consumption is more volatile than income and the trade balance and sovereign spread are countercyclical¹³.

Figures 24.6 and 24.7 illustrate why the predictions of the quantitative model are consistent with the data for emerging economies. In Figure 24.6, as in the two-period model, the bond price decreases with debt. The bond price also increases with income, implying that the sovereign's borrowing set shrinks when income drops, precisely when the sovereign needs borrowing the most. This occurs because when income takes low values, investors anticipate lower income and thus stronger default incentives in future periods.¹⁴ The solid dots in Figure 24.6 illustrate how, in equilibrium, the sovereign chooses to issue fewer bonds and at a lower price when the income realization is low (one standard deviation below the mean in the graph). Figure 24.7 shows that when income is low, because of the lower borrowing, consumption drops more than income, accounting for the excess consumption volatility in emerging economies.

¹²As discussed in the Appendix, there are two important assumptions for achieving this: (i) the government issues long-term debt (Arellano and Ramanarayanan, 2012; Chatterjee and Eyigungor, 2012; Hatchondo and Martinez, 2009), and (ii) the cost of defaulting increases more than proportionally with income (Arellano, 2008; Chatterjee and Eyigungor, 2012).

¹³Note that Mexico has experienced a moderation in aggregate fluctuations in recent years and thus consumption is as volatile as income in Table 24.3 but it was 24% more volatile than income in the earlier sample used by Aguiar and Gopinath, 2006.

¹⁴In terms of the two-period model, consider a version of this model with $Y_1 > 0$ and a c.d.f. function for period-2 income that is decreasing with respect to period-1 income, $F(\cdot | Y_1)$. For any chosen B_2 , a higher income in the first period shifts the distribution of future income (Y_2) to the right, thereby reducing the default probability. As a consequence, the equilibrium bond price

$$q_1(B_2, Y_1) = 1 - F(B_2/\phi | Y_1)$$

is an increasing function of period-1 income.

Table 24.3: Business Cycle Statistics: Model and Data

Targeted moments		
	Model	Data
Mean Debt-to-GDP	43	43
Mean <i>Spread</i>	3.2	3.2
Non-Targeted moments		
$\sigma(C)/\sigma(Y)$	1.3	1.0
$\sigma(TB/Y)$	0.8	1.3
$\sigma(Spread)$	1.6	2.2
$\rho(TB/Y, Y)$	-0.7	-0.6
$\rho(C, Y)$	0.98	0.91
$\rho(Spread, Y)$	-0.8	-0.4
$\rho(Spread, TB/Y)$	0.9	0.6

Note: The standard deviation of x is denoted by $\sigma(x)$. The coefficient of correlation between x and z is denoted by $\rho(x, z)$. Moments are computed using detrended series. Trends are computed using the Hodrick-Prescott filter with a smoothing parameter of 1,600. Moments for the simulations correspond to the mean value of each moment in 500 simulation samples. We take the last 120 periods (30 years) for each sample without a default episode. Simulation samples start at least five years after a default. Default episodes are excluded to improve comparability with the data. Consumption and income are expressed in logs.

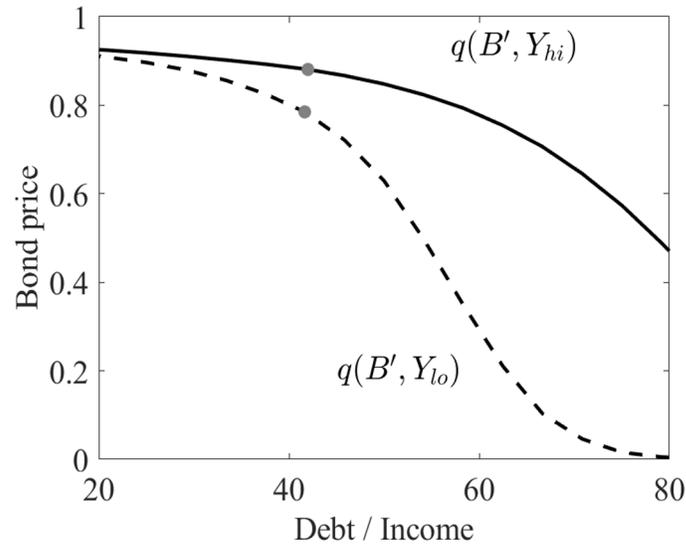


Figure 24.6: Bond price schedule for a low income ($Y_{lo} = E(Y) - \sigma(Y)$) and a high income level ($Y_{hi} = E(Y) + \sigma(Y)$).

Notes: The horizontal axis displays the ratio of debt to average annual income $B'/4E(Y)$ the government could choose. Solid dots correspond to the optimal choices when the government enters the period with the mean debt level in the simulations.

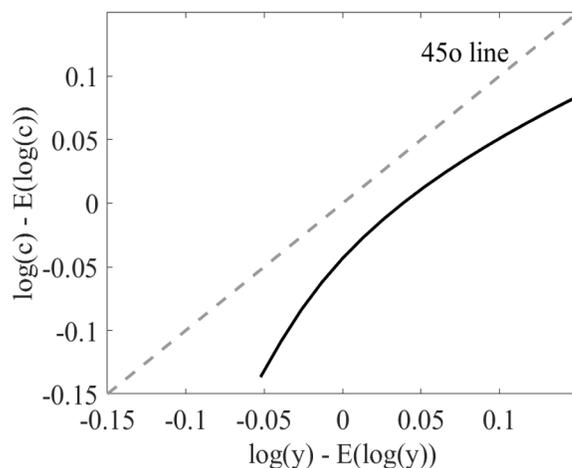


Figure 24.7: Consumption policy under repayment when the sovereign enters the period with the mean debt level in the simulations.

Normative implications in economies with default risk. We have shown that the canonical default model offers an empirically plausible micro-founded account of business cycle dynamics in emerging economies. Could this model also be useful for policy design? Note that the implication of the model is that a benevolent government (maximizing the utility of the representative consumer) chooses to face high sovereign risk. At the same time, as discussed in the introduction, having high sovereign risk seems to have significant adverse effects not only on business cycle dynamics but also on the ability of governments to react to large shocks (e.g., COVID-19) and on investment and growth. Given these costs of sovereign risk, why would benevolent governments still choose debt levels that command high default risk?

The answer to this question may be related to time inconsistency problems that for example, also lead governments to choose high inflation. The default model features two time inconsistency problems. First, the government may want to commit to default less in the future (in comparison to how often it chooses to default in equilibrium). The government acting in period t could benefit from restricting defaults in period $t + 1$ (and with long-term debt, in every future period) because this would expand the borrowing set in t (i.e., would increase bond prices in t). An improved borrowing set would allow the government to better exploit the gains from trade derived from its relative impatience as the calibration of quantitative default models typically assumes that the government discounts the future more than lenders. With β being the government's discount factor and r be the interest rate representing the lenders' opportunity cost of lending, the standard calibration assumes that $\beta(1 + r) < 1$. Bolton and Jeanne (2009) study a model in which issuing debt that is more costly to restructure can be optimal, as this would deter future defaults. As mentioned before, governments in emerging countries do issue debt in foreign financial centers and include covenants in debt contracts to make their debt more difficult to restructure. Mateos-Planas et al. (2025) find that for standard calibrations of the default model, if the government could commit to its future default policy, it would commit to not defaulting. They also show that for less standard calibrations it is possible to obtain default under commitment, but

default is very rare. This is, the time inconsistency problem due to the government's inability to commit to future default decisions accounts for the majority if not all equilibrium sovereign risk. [Hatchondo et al. \(2022\)](#) show that committing to a no-default rule would generate large welfare gains for the government, but the temptation to break this rule would also be large.

The second time inconsistency problem stems from the government's inability to commit to future borrowing levels. This problem only arises when we assume that the government can issue long-term debt ([Arellano and Ramanarayanan, 2012](#); [Chatterjee and Eyigungor, 2012](#); [Hatchondo and Martinez, 2009](#)). The mechanism behind this time consistency problem is similar to the one described above: by committing to lower future borrowing levels, the government could increase current bond prices. For example, by restricting borrowing in $t + 1$, the sovereign can reduce default probabilities in $t + 2$ (and after) and expand the borrowing set in t . Note that with one-period debt, the default probability in $t + 2$ does not affect bond prices in t (which only depend on the default probability in $t + 1$) and thus, this time inconsistency problem does not arise (see [Aguiar and Amador, 2019](#) for a formal proof).

Assuming differentiability and that the government issues perpetuities with coupon payments declining at the rate δ , the time-inconsistency problem in borrowing policies is apparent from the period- t optimality condition:

$$u'(C_t) \left[q_t + \frac{\partial q_t}{\partial B_{t+1}} \left[\underbrace{B_{t+1} - (1 - \delta)B_t}_{\text{Bonds issued at } t} \right] \right] = \beta \mathbb{E} [V_1(B_{t+1}, Y_{t+1}) | Y_t], \quad (24.13)$$

where V_1 denotes the derivative of the government's value function with respect to debt. Note that the optimality condition in equation (24.13) resembles the one for the two-period model in equation (24.6).

Equation (24.13) illustrates how, when the sovereign borrows in t , it takes into account the negative effect that an extra bond sold has on the price at which it sells bonds, q_t . However, the sovereign deciding in t does not take into account how borrowing in t affects the price of the bonds it issued in periods before t . Of course, a government deciding in period t should not take into account how its borrowing affect the price of bonds it issued in previous periods. However, a government that could commit to an optimal borrowing plan before t , would take into account how borrowing at t would affect the price of bonds (and thus utility) before t (as the government who could commit to future default plans would take into account the effect of future default on past prices).

As discussed in the introduction, a large and increasing number of countries are implementing fiscal rules to limit borrowing by future governments. [Hatchondo et al. \(2022\)](#) use the default model to discuss the design of fiscal rules. [Chatterjee and Eyigungor \(2015\)](#) and [Hatchondo, Martinez, and Sosa Padilla \(2016\)](#) show how alternative bond contracts could help mitigate the time inconsistency problem in borrowing policies.