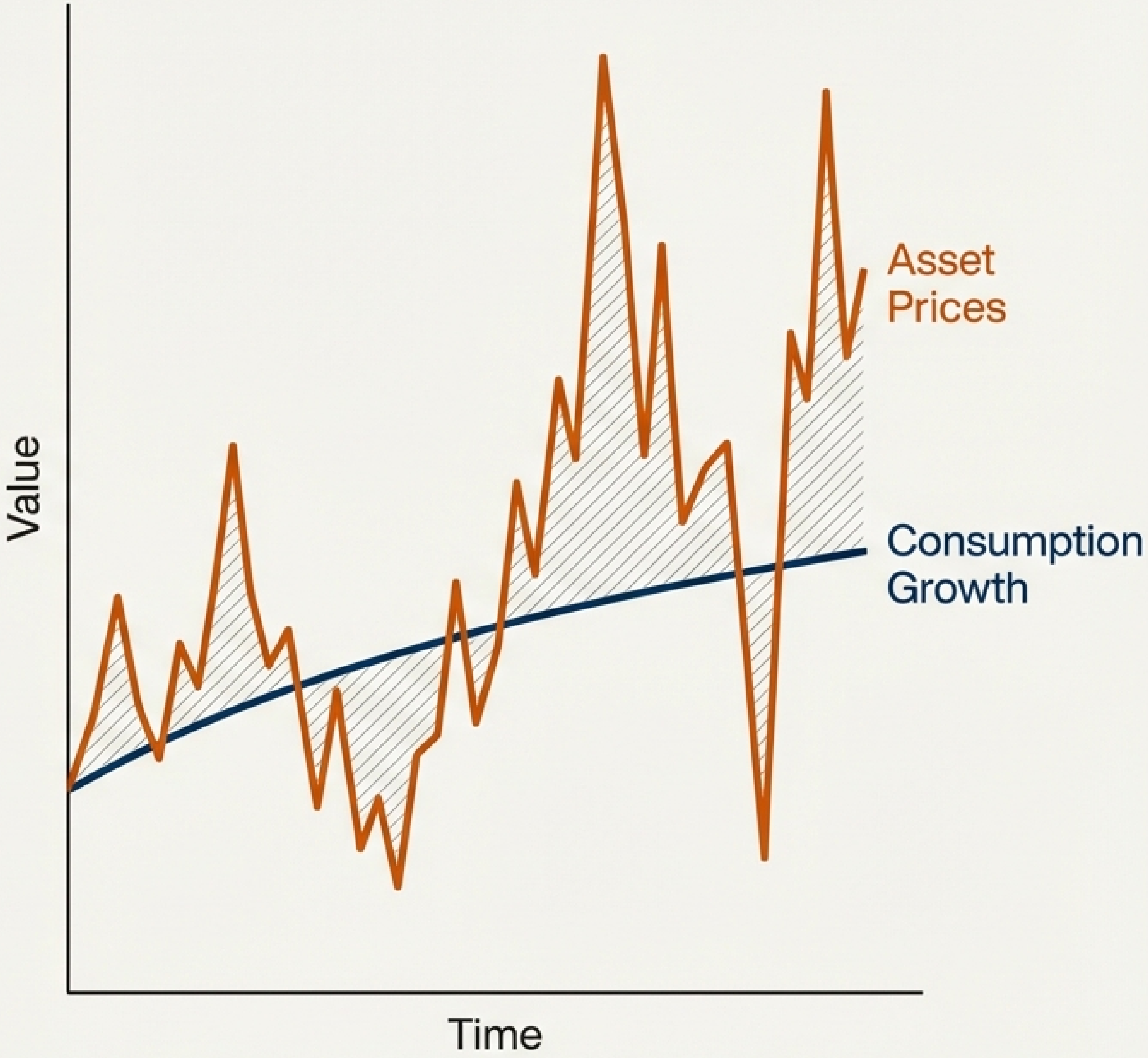
The Valuation of Risky Assets

From Neoclassical Theory to the Equity Premium Puzzle



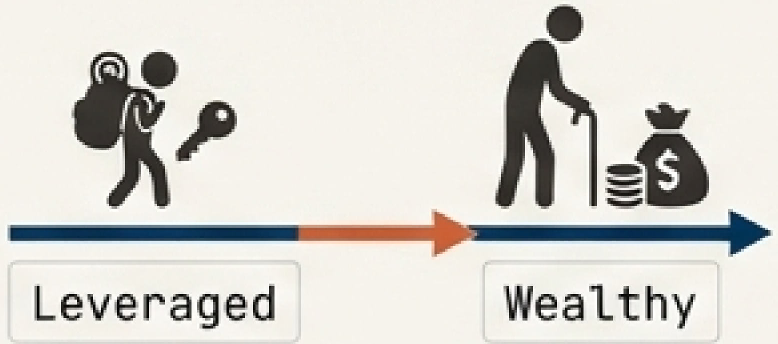
Based on 'Asset Prices' by Monika Piazzesi and Martin Schneider (Chapter 16)

Why We Move Beyond the Single-Asset Benchmark

Heterogeneity

Households hold vastly different portfolios. In the US, 65% own houses, but only 52% own stocks.

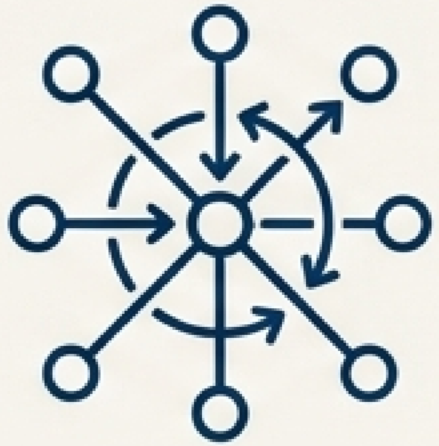
Agents are not identical.



Lifecycle Differences

Young households are leveraged (mortgages). Older households accumulate wealth. Asset

price fluctuations impact welfare differently by age.



Market Structure

Multiple assets allow us to test for "Complete Markets." Do agents have enough instruments to insure each other against all risks?

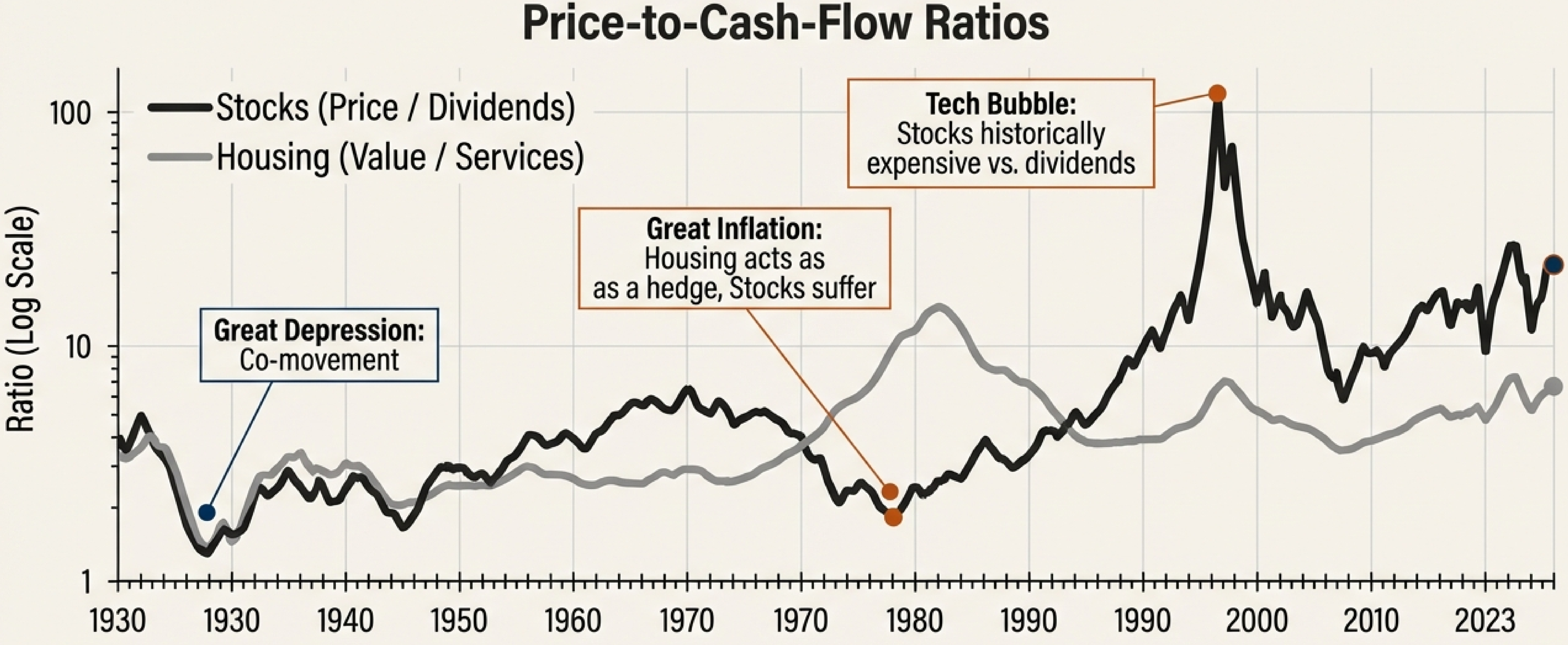
Global Context: Pensions

Pension systems drive participation.

US plans force active allocation.

European government-funded pensions often reduce household portfolio choices.

The Decoupling of Prices and Cash Flows (1930–2023)



Asset values are highly volatile relative to the cash flows they produce. Prices are driven by more than just current earnings.

NotebookLM

The Framework: A Dynamic Stochastic Economy

The Physical Environment

Time & Uncertainty
Infinite horizon. State ω_t realized at each date t.

→

The Agents
Maximize Expected Utility $U_i(c_i)$.

→

Endowments
Agents receive goods y_i in different states.

The Theoretical Benchmark: The Planner

The Central Planner maximizes weighted sum of utilities:

$$\max \sum \lambda_i U_i(c_i)$$

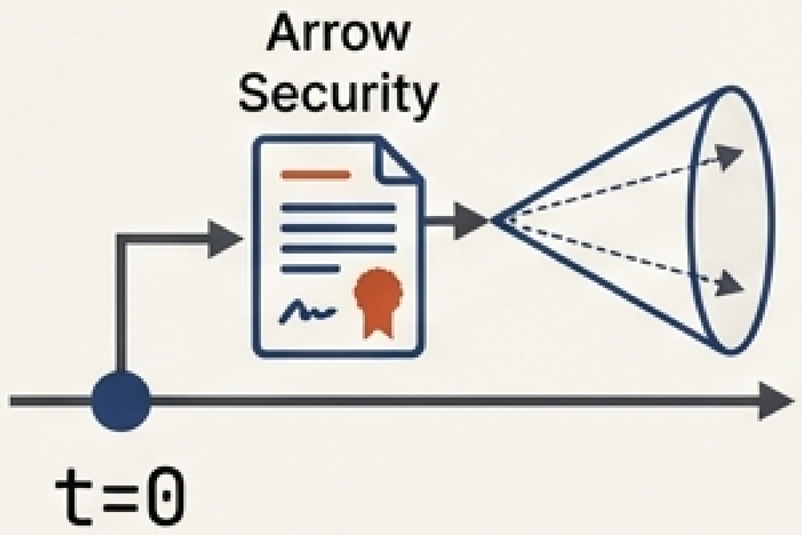
First Order Condition:

$$\text{Weighted Marginal Utility} = \text{Lagrange Multiplier } \mu_t(\omega_t)$$

Result: In an efficient allocation, Marginal Rates of Substitution are equalized across all agents.

Market Structure: Arrow-Debreu vs. Sequential Trading

Arrow-Debreu (Time 0 Trading)

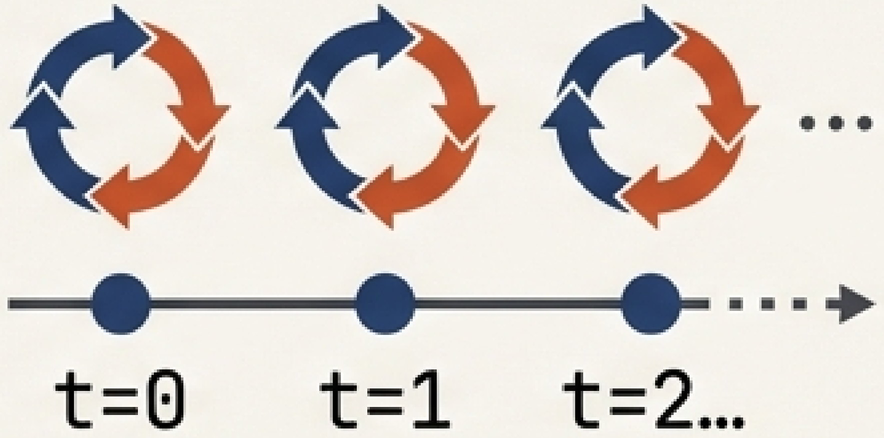


Trade ONCE at Time 0 for all future contingencies.

Requires perfect foresight and trust.

Price: p_0

Sequential Markets (Spot Trading)



Trade assets (stocks/bonds) spot-by-spot at every date.

Agents rebalance portfolios constantly.

Price: p_t

The Equivalence Theorem

If Markets are COMPLETE (Assets = States), these two structures yield identical allocations.

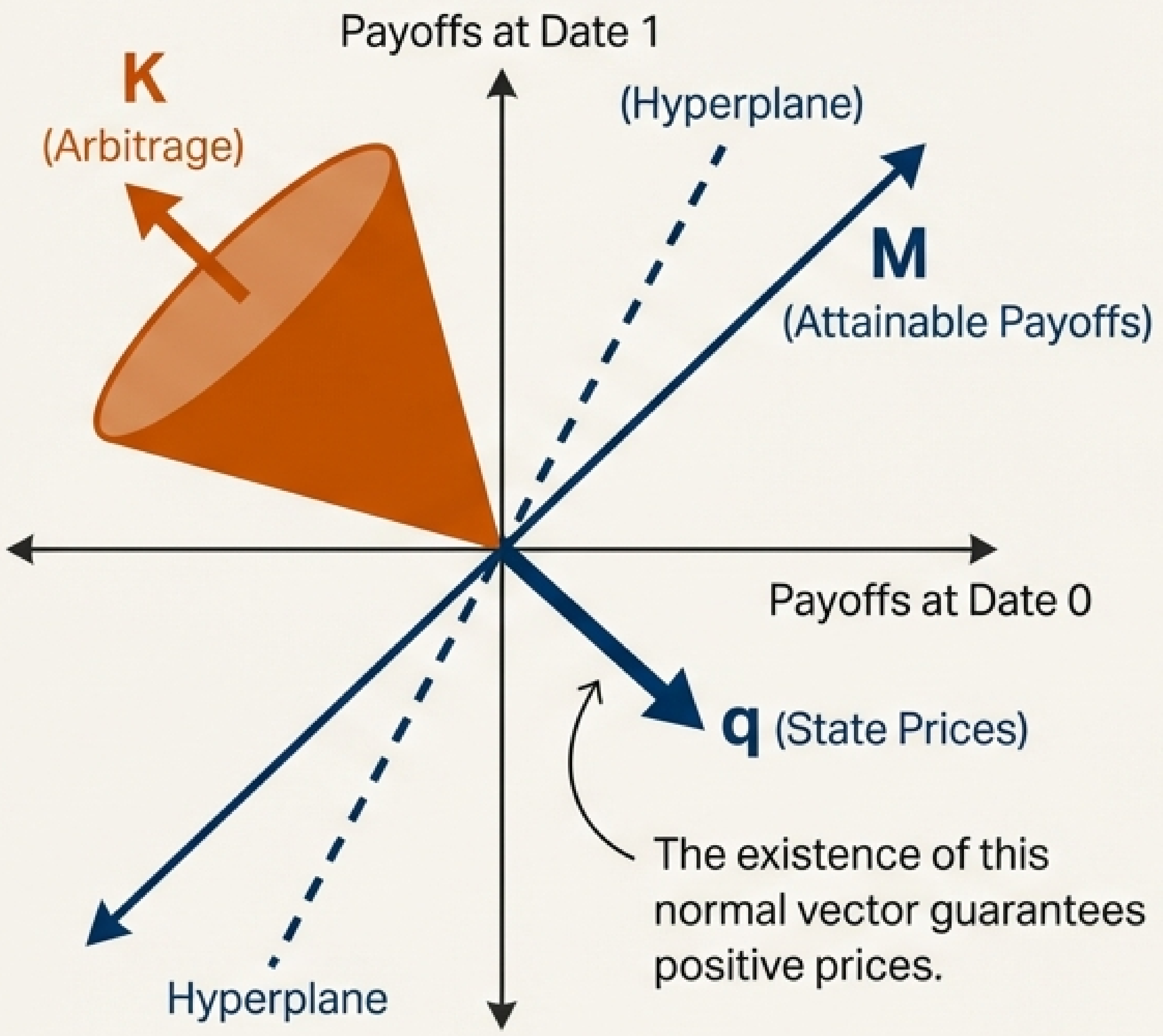
The Geometry of 'No Arbitrage'

Definition: An arbitrage is a portfolio costing zero today with positive payoffs tomorrow.

Fundamental Theorem of Asset Pricing:

$$\boxed{\text{No Arbitrage} \Leftrightarrow \text{Existence of strictly positive State Prices (q)}}$$

$$p = D^T q$$



State Prices and the 'Hunger' Index

$$p_n = E\,[\,d_n(\omega)\,M(\omega)\,]$$

Asset Price = Expected Covariance of Payoff (d) and Hunger (M)

High M (Hungry)

- State of Scarcity / Recession.
- Payoffs here are highly valuable (Insurance).
- M(ω) is High.

Low M (Full)

- State of Abundance / Boom.
- Payoffs here are worth less.
- M(ω) is Low.

Subjectivity:

In incomplete markets, agents may disagree on state prices (q) but agree on asset prices (p).

From Risk to Risk Premium

Variance is not the measure of risk. Covariance with the SDF is what matters.

$$E[R] - R_f = \frac{-Cov(R,\ M)}{E[M]}$$

Asset Type	Covariance with M	Pays Off When...	Resulting Premium
Insurance Asset	Positive (+)	Agents are "Hungry" (High M)	Negative Premium (You pay to hold it)
Risky Asset (Stocks)	Negative (-)	Agents are "Full" (Low M)	Positive Equity Premium (You demand pay to hold it)

The Model Under the Microscope: Power Utility

$$U(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

How much agents fear uncertainty.

Controls the Equity Premium.

How much agents resist moving consumption over time.

Controls the Risk-Free Rate.

1. **Smoothing Motive:** High growth leads to borrowing demand → High Interest Rates.
2. **Precautionary Motive:** High volatility leads to saving demand → Low Interest Rates.

The Consumption-CAPM Formula

Deriving the Testable Equation

Assumption: Consumption and Returns are Jointly Lognormal.

The Formula:

$$\underbrace{\text{Equity Premium}}_{\text{The Data we see}} \approx \underbrace{\gamma}_{\substack{\text{Price of Risk} \\ \text{(Unknown)}}} \times \underbrace{\text{Cov(Consumption, Returns)}}_{\substack{\text{Quantity of Risk} \\ \text{(Data)}}}$$

$$\text{Equity Premium} \approx \gamma \times \sigma_c \times \sigma_r \times \text{correlation}$$

Goal: We plug in historical data for the Premium and Covariance to solve for γ.

The Empirical Baseline (US Data 1929-2024)

Variable	Mean	Volatility (Std Dev)	
Consumption Growth	1.75%	2.33%	Very Smooth
Risk-Free Rate	0.74%	3.80%	Low and Stable
Stock Returns (S&P 500)	8.28%	18.36%	Volatile

The Target Gap:

Equity Premium = E[Stocks] – E[Risk-Free]

Equity Premium ≈ 7.5%

Puzzle 1: The Equity Premium Puzzle

The Equation Crash

The Equation
(Navy Blue, Crimson Pro)

$$0.075 = \gamma \times 0.0043$$

(Where 0.075 is the Premium, and 0.0043 is the covariance $\sigma_c \times \sigma_r$)

The Result
(Navy Blue, Crimson Pro)

$$\gamma \approx 17$$

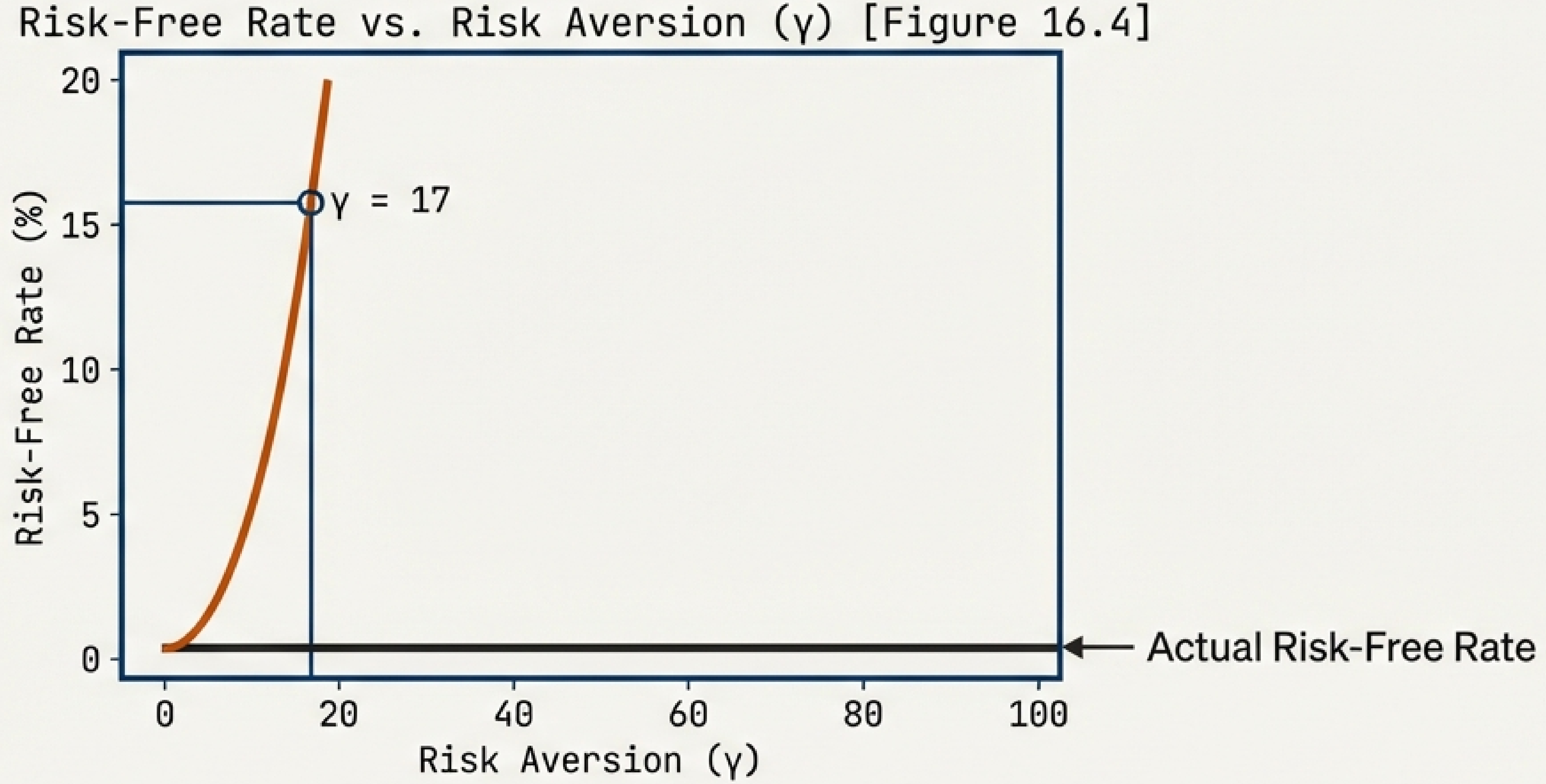
Analysis Text

- **Economic Intuition:** Economists expect γ to be between 1 and 5.
- **Implication:** A γ of 17 implies extreme paranoia. An agent with γ=17 would refuse a gamble with excellent odds simply to avoid small variances.

The Puzzle: Consumption is too smooth (low quantity of risk) to justify the massive premium investors demand. The model fails.

Puzzle 2: The Risk-Free Rate Puzzle

Why can't we just accept $\gamma = 17$? Because it breaks the bond market.



Intertemporal Smoothing Motive: If γ is high, agents hate uneven growth. They try to borrow against future growth, driving interest rates to absurd levels (>15%).

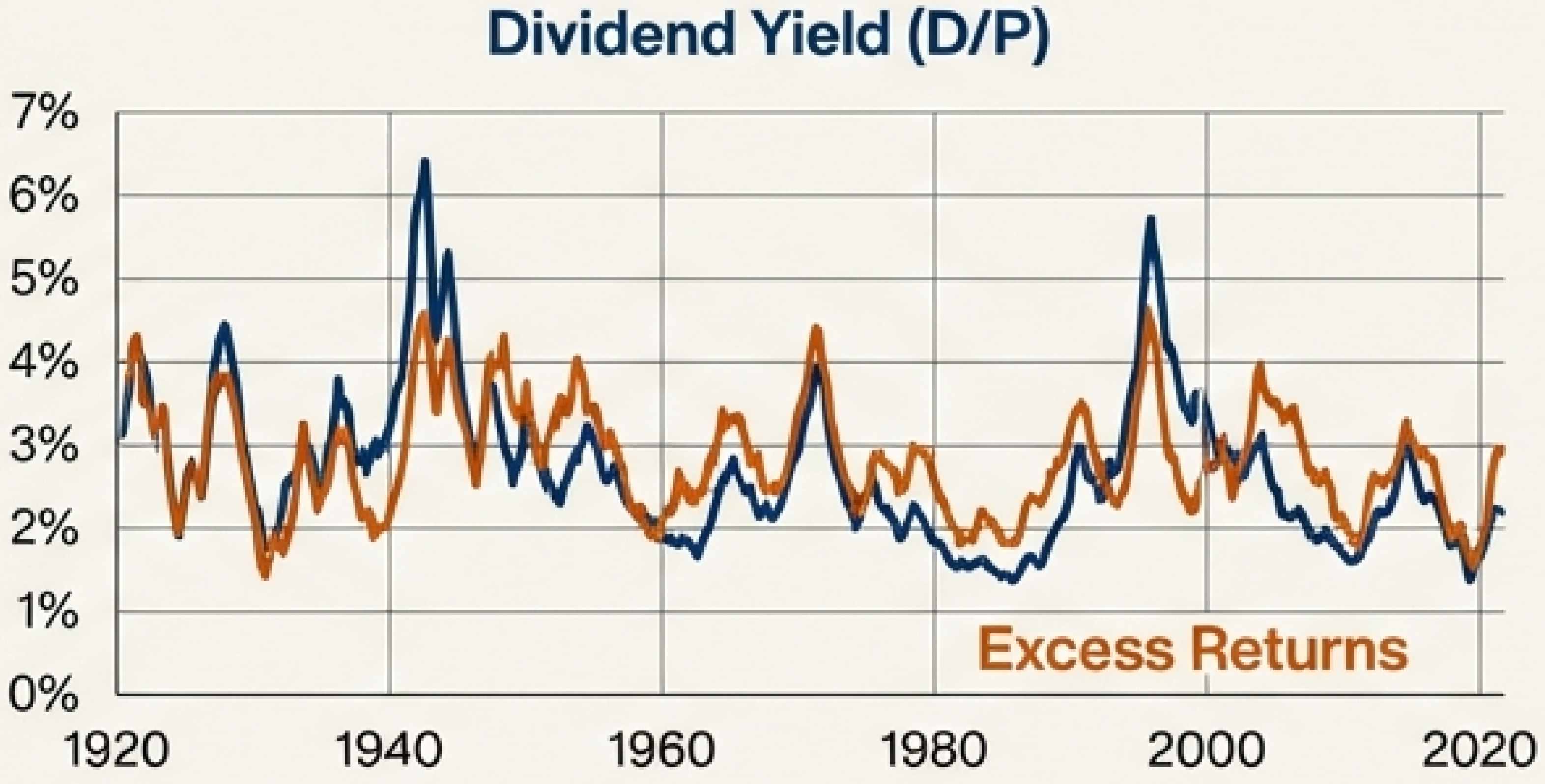
Puzzle 3: The Excess Volatility Puzzle

The Theory

If consumption growth is random walk (i.i.d.), Price/Dividend ratios should be constant.

$$\frac{p_t}{d_t} = \text{constant}$$

The Reality (Shiller's Insight)



1. Prices move far more than dividends.
2. High Dividend Yields predict high future returns.
3. Conclusion: Returns are predictable, contradicting the basic Random Walk theory.

The Path Forward: Fixing the Model

~~**Standard Power Utility Model**~~

~~Fails on Premium, Rate, and Volatility.~~

Epstein-Zin Utility

Decouples Risk Aversion (γ) from Intertemporal Substitution (EIS). Fixes the Risk-Free Rate.

Time-Varying Moments

Disaster Risk / Long-Run Risk models where volatility fluctuates over time.

Behavioral Finance

Sentiment, ambiguity aversion, and distorted beliefs.

"Asset prices are crucial moments that discipline our models. The failure of the standard model is not an end, but the beginning of better theory."