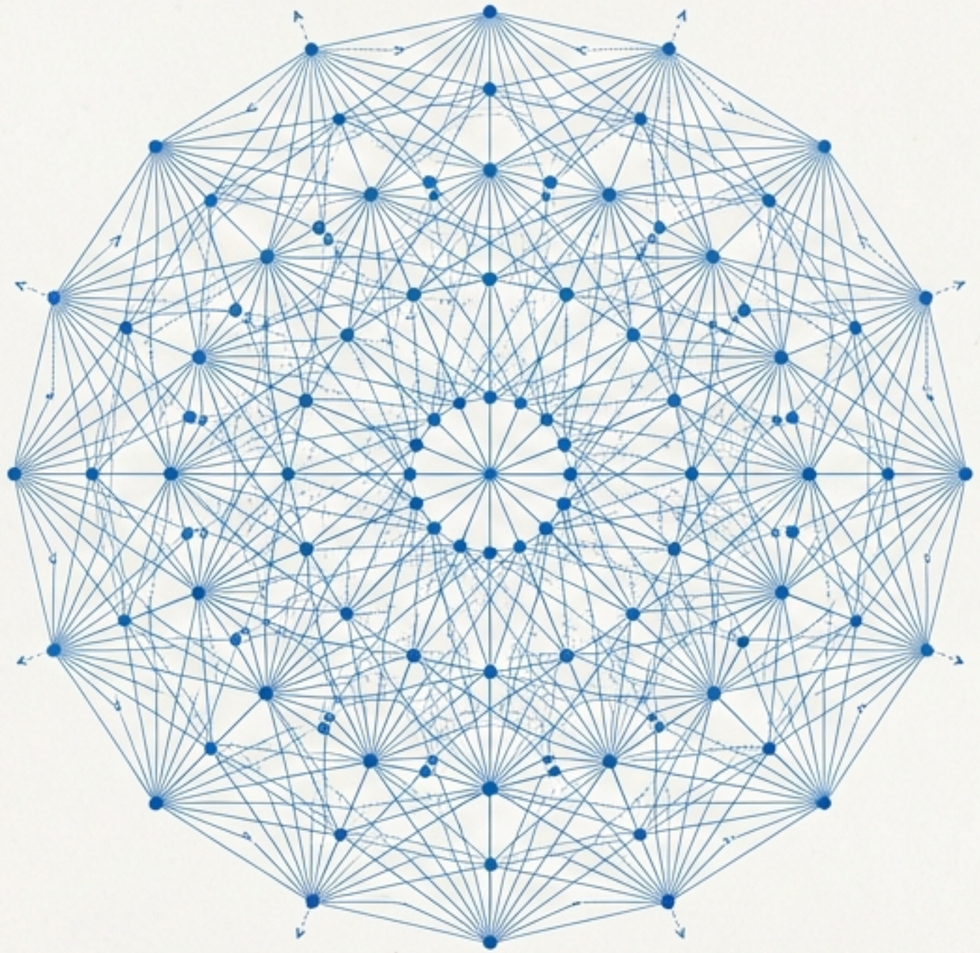
The Architect vs. The Reality

Welfare Economics, The Invisible Hand, and the Limits of Design.

The First Welfare Theorem acts as the architect's blueprint for a perfect economy—a theoretical ideal where markets coordinate disparate interests into social optimality. This deck explores how that blueprint functions, where structural cracks appear (Market Failures), and what happens when the building must last forever (Overlapping Generations).

Act I: The Blueprint

The Miracle of Coordination



The First Welfare Theorem (FWT):

A competitive equilibrium is Pareto optimal.

The Logic:

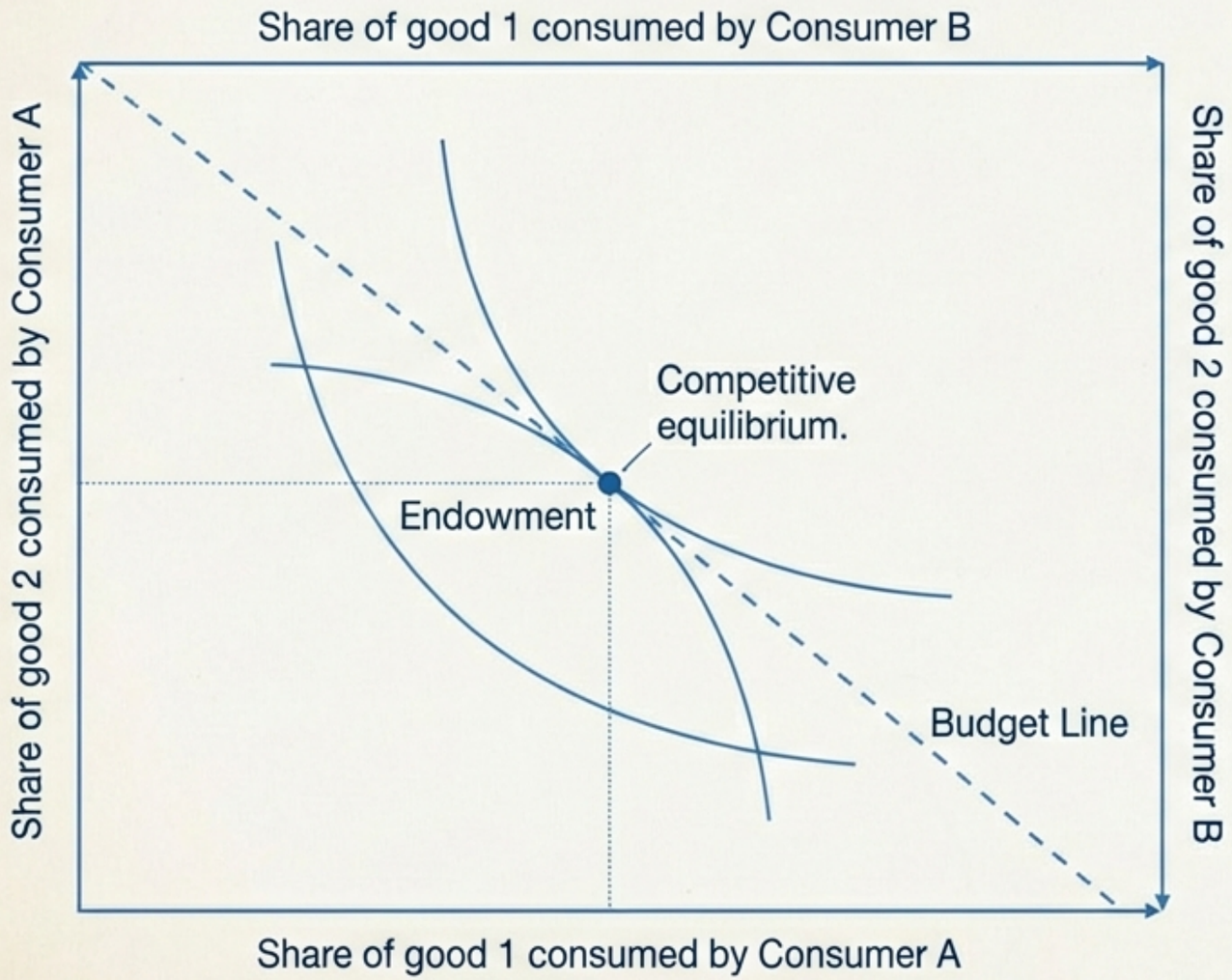
Markets coordinate the activities of large numbers of consumers and firms, separated across space and time, without a central commander. If a feasible allocation existed that everyone preferred, it would have been more expensive than their current choices—meaning it is unaffordable in aggregate.

The Condition:

Local Non-Satiation (LNS): The assumption that a consumer can always be made better off by an infinitesimally higher consumption of some good.

Adam Smith's Invisible Hand is mathematically rigorous, provided the assumptions hold.

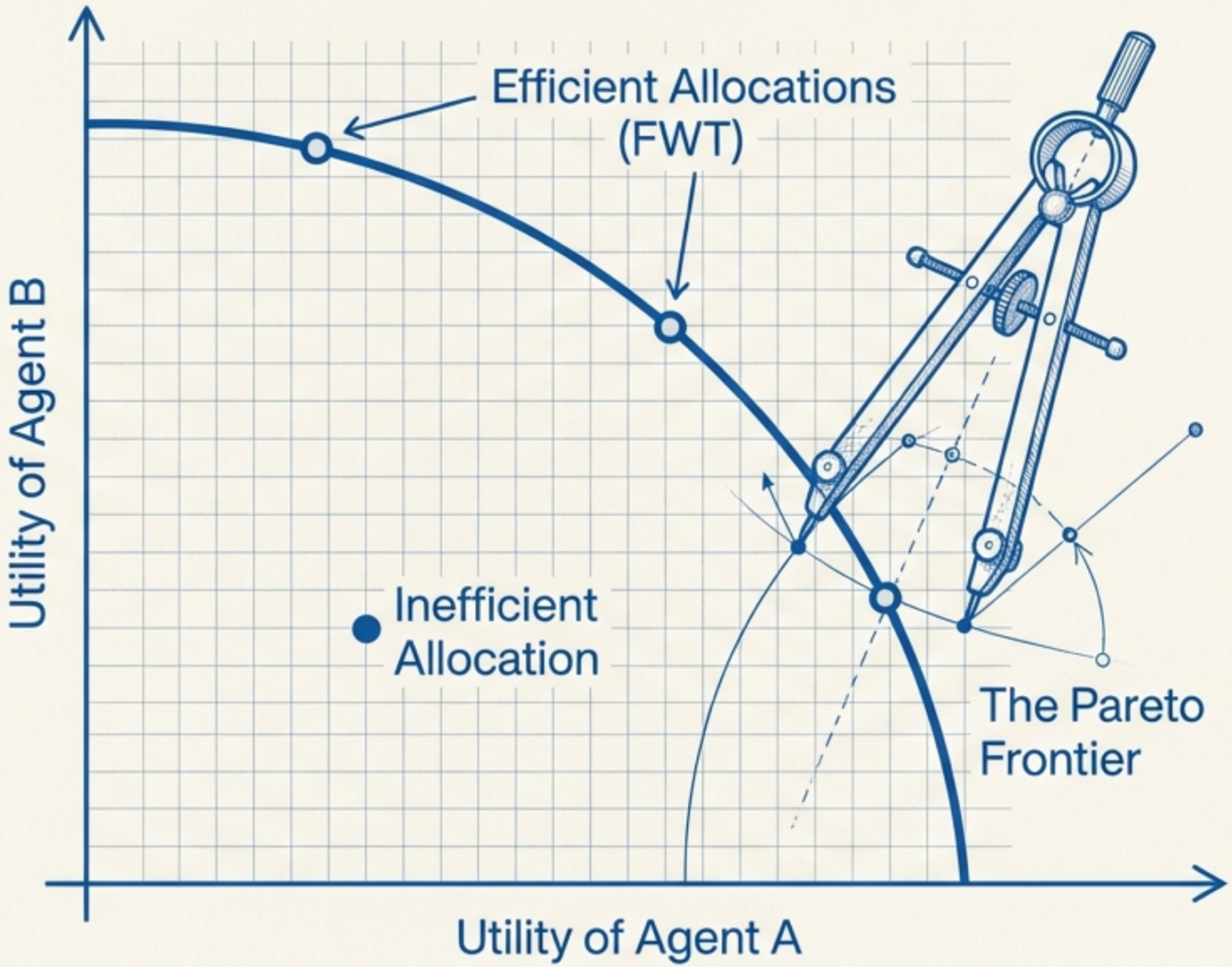
Visualizing Efficiency: The Geometry of Tangency



Geometric Intuition

1. **Tangency:** At the equilibrium point, the indifference curves of Consumer A and Consumer B kiss but do not cross. You cannot move Consumer A to a higher curve without pushing Consumer B to a lower one.
2. **MRS = Relative Prices:** The slope of the indifference curve (Marginal Rate of Substitution) equals the slope of the budget line (price ratio).
3. **Market Clearing:** The Invisible Hand forces the consumption bundles to coincide. What isn't consumed by A is consumed by B.

The Social Planner & The Pareto Frontier



The Social Planner

We can simulate Pareto optimal outcomes by imagining an omniscient Planner who maximizes a weighted sum of utilities.

Negishi Weights

The Planner assigns weights (μ) to each agent based on the inverse of their shadow value of wealth ($\mu = 1/\lambda$).
By varying these weights, the Planner traces the entire frontier.

Key Insight:

The market outcome is just one specific point on this frontier—usually the one weighting agents according to their initial wealth.

Act II: Structural Cracks

Static Market Failures

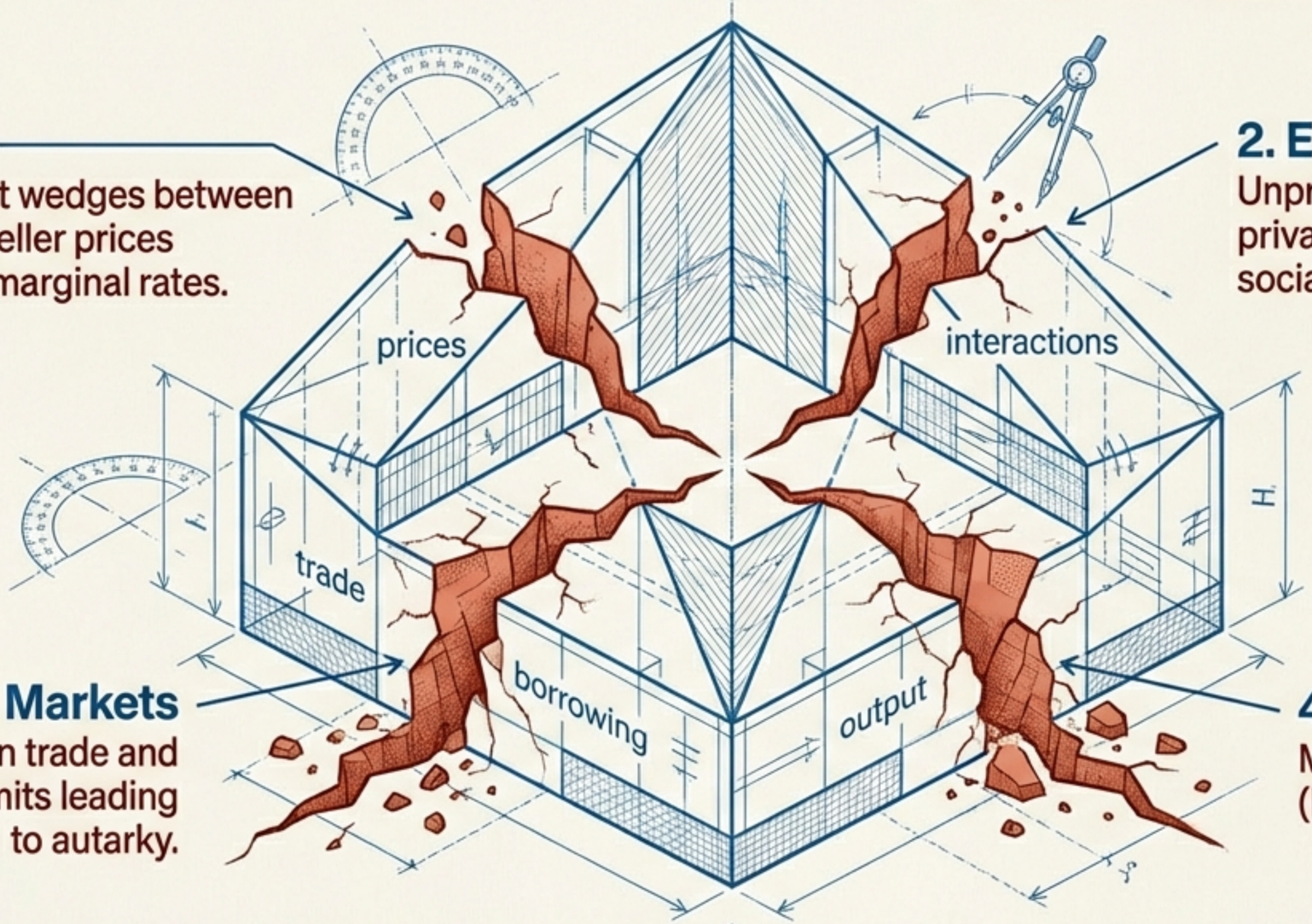
The First Welfare Theorem fails when its architectural assumptions are violated. We view these violations not just as math errors, but as structural cracks in the foundation of the economy.

1. Taxes

Government wedges between buyer and seller prices decoupled marginal rates.

2. Externalities

Unpriced interactions where private incentives ignore social costs.



3. Missing Markets

Constraints on trade and borrowing limits leading to autarky.

4. Market Power

Monopolistic price setting ($P > MC$) restricts output.

Crack #1: The Tax Wedge

The Mechanism

Taxes create a disconnect between the price the buyer pays and the price the seller receives.

The Breakdown

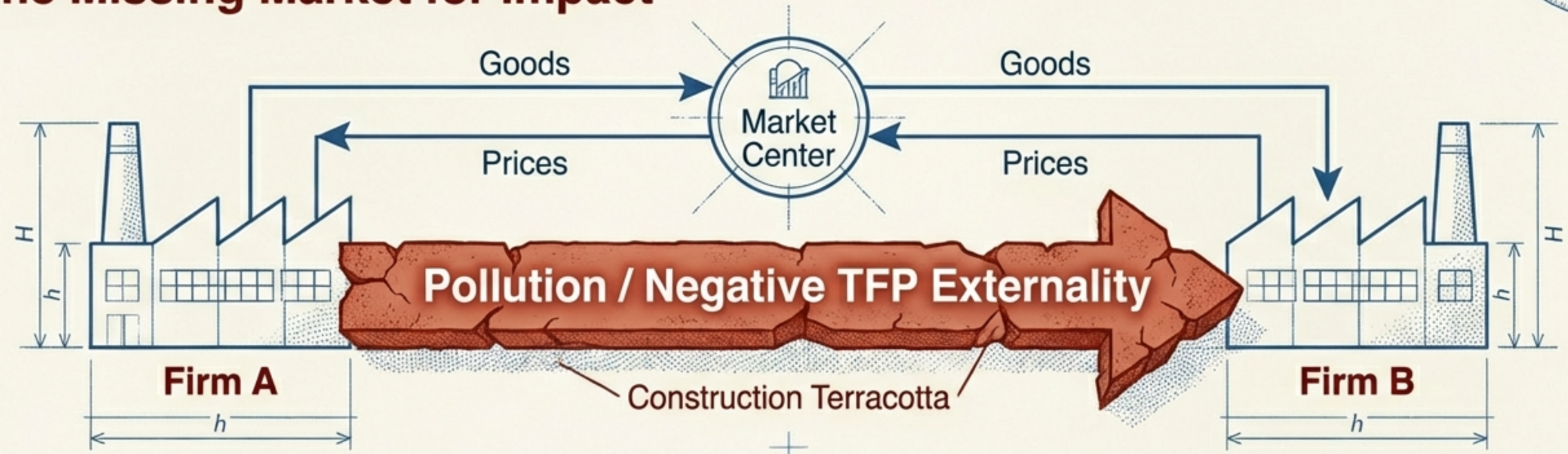
* **Lump-sum taxes:** Efficient. They just move the allocation *along* the Pareto frontier.
* **Distortionary taxes:** Inefficient. They push the allocation *off* the frontier.

The Euler Equation Failure

With capital taxes, the marginal rate of substitution between goods at time t and $t+1$ no longer equals the transformation rate.
Consumer faces: $(1 - \tau)r$
Firm faces: r

Crack #2: Externalities

The Missing Market for Impact



The Scenario

A negative externality where Firm A's production lowers Firm B's productivity: A(Y).

The Disconnect

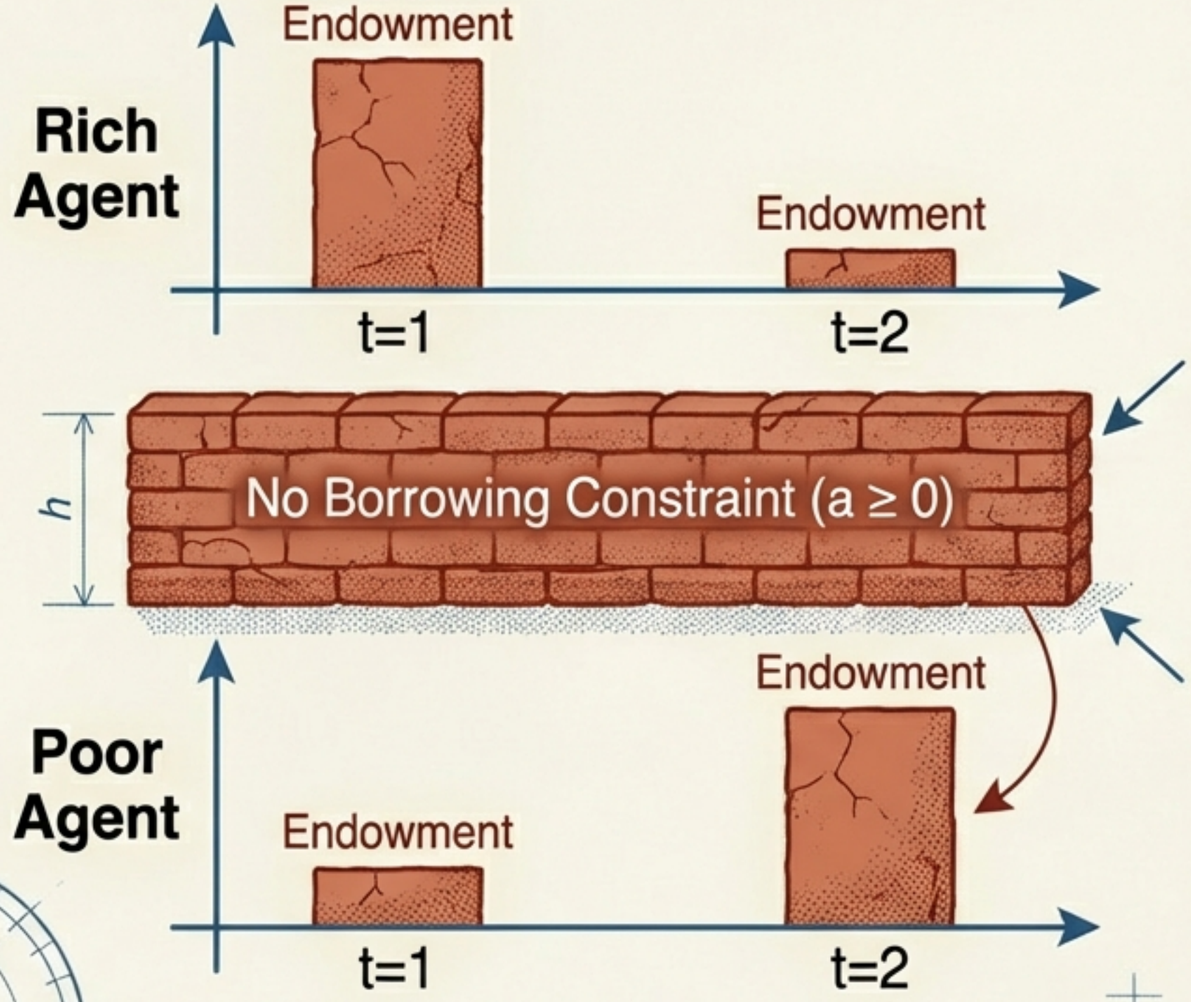
* **Competitive Equilibrium:** Firms ignore their impact on A. They produce until Marginal Benefit = Private Cost.
* **Social Planner:** Internalizes the damage. Recognizes that $A'(Y) < 0$.

Result

The market over-produces and over-works because it ignores the "social cost" of congestion and pollution.

Crack #3: Missing Markets

Borrowing Constraints & Pecuniary Externalities



The Outcome: Autarky

Without the ability to trade promises (borrowing), agents must consume exactly what they have. Consumption smoothing is impossible.

Price Distortion

Interest rates plummet to artificially low levels (or even negative) because the demand for savings evaporates.

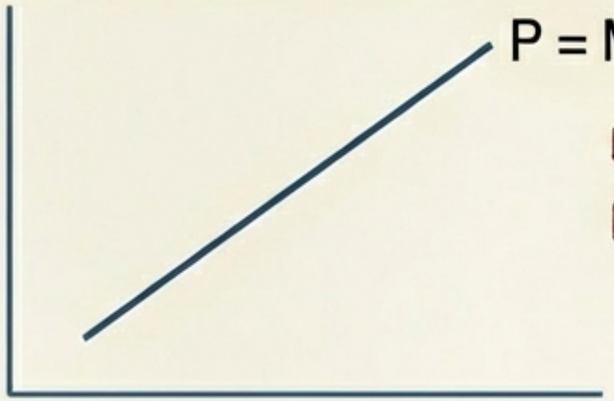
Pecuniary Externality

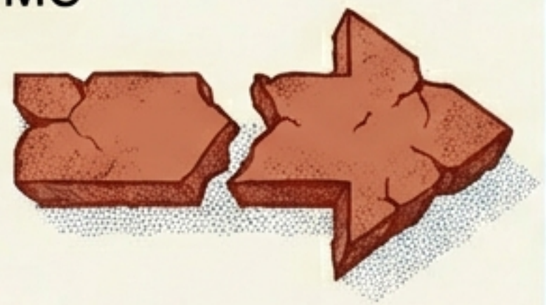
Agents don't realize how their consumption choices affect prices (**q**), which in turn tightens the borrowing constraints of others. The consumption possibility set becomes price-dependent.

Crack #4: Market Power

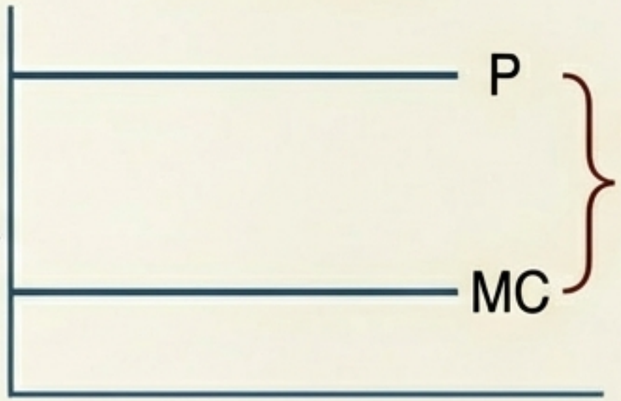
Monopolistic Competition & Markups

Perfect Competition





Monopoly



The Distortion

Firms with market power charge a markup ($\mu > 1$) over marginal cost.

Markup formula: $\mu = \varepsilon / (\varepsilon - 1)$.

The Inefficiency

To maintain this high price, firms must restrict output.

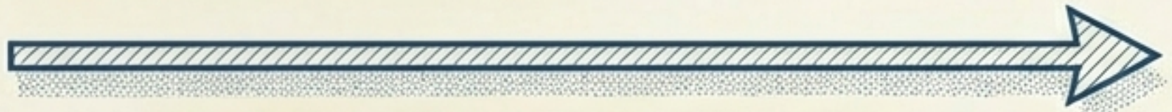
* **Equilibrium:** $A(1 - 1/\varepsilon) = C\, v'(C/A)$.
* **Efficient:** $A = C\, v'(C/A)$

Conclusion: Labor and output are too low. The economy operates *inside* the production possibility frontier.

Act III: The Infinite Corridor

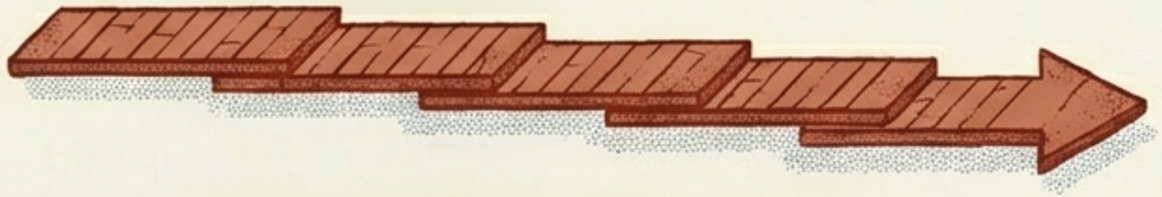
Overlapping Generations (OG) and Dynamic Inefficiency

Dynastic Model



Infinite Life, Finite Budget

Overlapping Generations



Finite Life, Infinite Time

The Plot Twist

Even without the frictions of Act II (taxes, monopolies), the system can fail simply because time is infinite and people are not.

The Paradox

In the standard FWT proof, the value of total resources must be finite. In OG models, new people are born forever, bringing new endowments. The sum of budgets across infinite agents might not converge.

The Litmus Test: Balasko-Shell Theorem

When is an Infinite Market Efficient?

Pareto Optimal IF AND ONLY IF: $\sum \frac{1}{p} = \infty$

Case A: Inefficient	Case B: Efficient
High Interest Rates (Prices fall fast). Sum is Finite (e.g., 1 + 1/3 + 1/9...).	Low Interest Rates (Prices stay high). Sum is Infinite (e.g., 1 + 1 + 1...).
Result: The market fails.	**Result:** The market works.

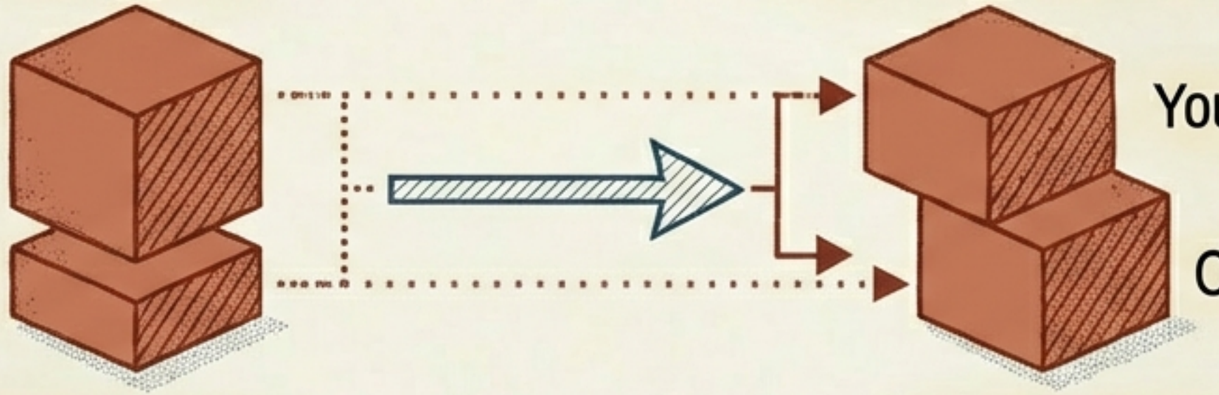
This theorem is the structural engineer's stress test for OG models.

Dynamic Inefficiency: The Free Lunch

Autarky

Young: 3 units

Old: 1 unit

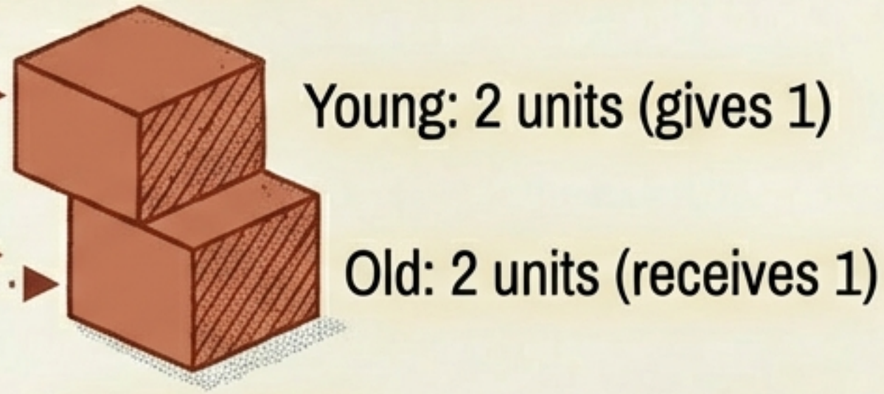


⚠ Utility: Low

Pay-As-You-Go Transfer

Young: 2 units (gives 1)

Old: 2 units (receives 1)



▲ Utility: High

The Mechanism

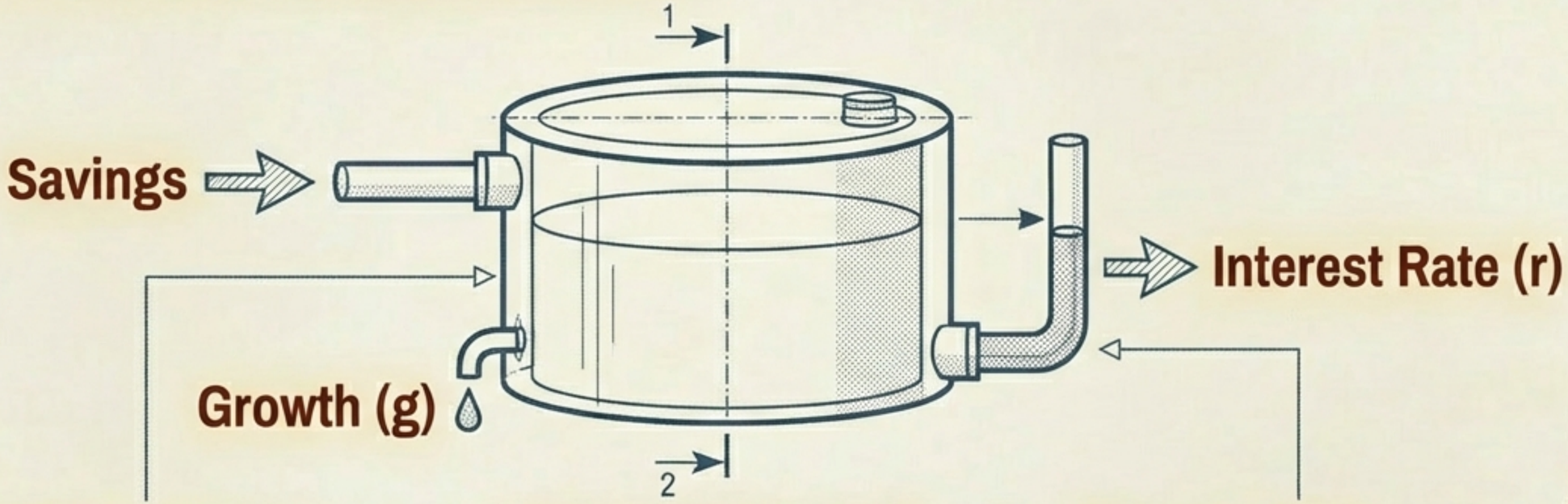
Take 1 unit from the Young, give to the Old.
Allocation becomes (2, 2).
Log(2) + Log(2) > Log(3) + Log(1).

The Insight

The initial old get a free unit. Everyone else is happier. This is possible because the economy was **Oversaving**. The market failed to coordinate the necessary transfer across generations.

Intertemporal Production & Capital

The Golden Rule of Accumulation



The Storage Trap

Even with capital storage, inefficiency is possible. If you store 1 unit and get only 1 back (return = 1), you are inefficient compared to a transfer system.

The Neoclassical Condition

Efficiency requires the Net Interest Rate ≥ Growth Rate.

$$r \geq g$$

Visual Logic

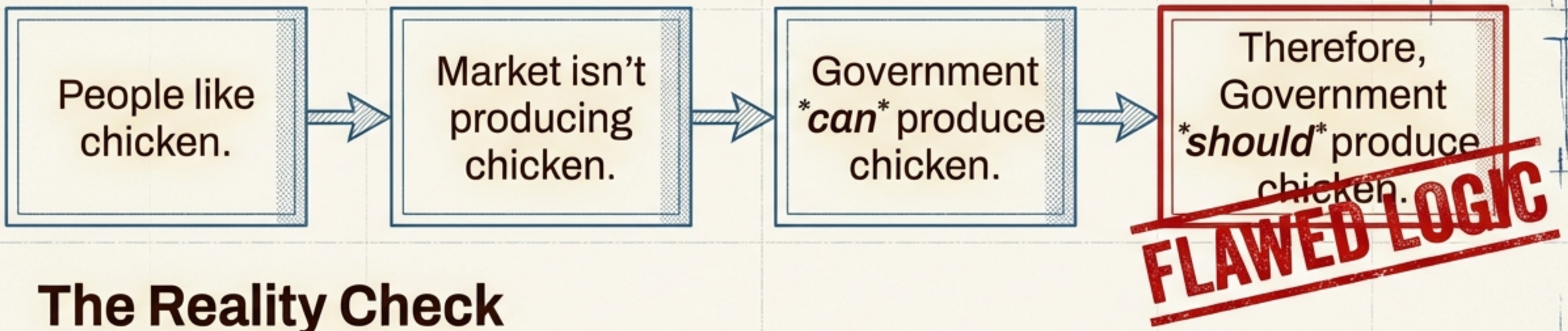
If r < g, the economy is accumulating 'too much' capital. We could consume some of that capital today and *still* maintain consumption forever. A true free lunch.

Act IV: Renovation & Repair

Government Policy as the Maintenance Crew

Problem	Tool
Externalities	**Pigouvian Taxes** Price the unpriced. Tax pollution to internalize the social cost.
Market Power	**Regulation / Subsidies** Anti-trust laws to break monopolies or subsidies to restore P=MC.
Missing Markets	**Social Insurance** Unemployment benefits and redistribution to mimic consumption smoothing.
Dynamic Inefficiency	**Social Security** A Pay-As-You-Go system creates the missing link between generations, transferring from Young to Old to correct oversaving.

Conclusion: The ‘Chicken Model" Warning



The Reality Check

Identifying a failure doesn’t guarantee the government is competent enough to fix it.

1. **Information Problems:** Can we calculate the optimal tax?
2. **Commitment Issues:** Will the next government honor the promise?
3. **Side Effects:** Does insurance cause moral hazard?

“We must analyze not just where the Architect’s blueprint failed, but whether the Maintenance Crew has the tools to fix it without causing more damage.”