Chapter 13: Growth

Graduate Macroeconomics Slides

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Outline

Chapter Overview

This chapter focuses on:

- Long-run economic growth and its determinants
- Empirical patterns of growth and development
- Neoclassical growth models, including investment-specific technical change
- Endogenous growth theories (Romer, quality ladders, creative destruction)
- Policy, welfare, and cross-country implications of growth models

"Understanding growth is of first-order importance to human welfare."

Why Study Growth?

- Over the last two centuries, advanced economies (e.g., U.S.) have experienced **steady growth in GDP per capita** of around 2% per year.
- The impacts of growth extend beyond GDP to improvements in living standards, technology, longevity, and welfare.
- Large disparities exist in income levels *across* countries.
- Key question: *Why are some countries rich and others poor? What drives sustained growth?*
- (Lucas Jr, 1988) quote: *"...Once one starts to think about them, it is hard to think about anything else."*

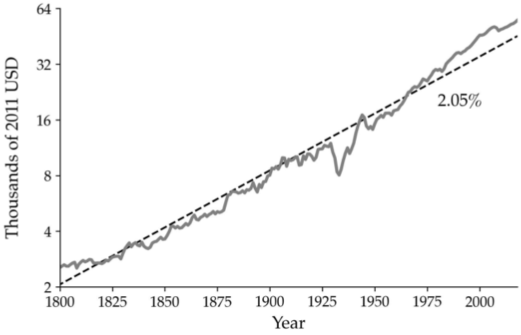

Figure 13.1: U.S. Real GDP per Capita

Figure: Source: (Maddison Project Database, 2020)

Historical Growth in the U.S.

Figure 13.1 shows:

- U.S. real GDP per capita over two centuries.
- Remarkably stable growth rate of slightly above 2% per year.
- Growth took off after the Industrial Revolution and has continued.

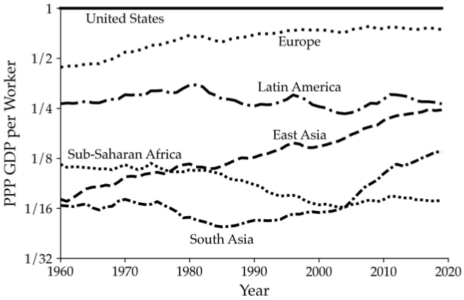

Figure 13.2: PPP GDP per Worker (U.S. = 1)

Figure: Source: (Feenstra et al., 2019)

Cross-Country Growth Dynamics

Figure 13.2 shows:

- GDP per worker for various regions relative to the U.S. (1960–2019).
- *Heterogeneity* in growth experiences:
  - Europe converged to U.S. levels until 1990, then grew in parallel.
  - Latin America mostly *sideways*.
  - East Asia (China, Japan, Korea) rapidly converged from low levels.
  - Sub-Saharan Africa and South Asia also show interesting patterns of partial catch-up and setbacks.

Key Conclusion: Most regions *grew* along with the U.S., but at **different rates**.

Distribution of World Income

- Global income distribution has shifted *to the right* over time, partly due to the rapid growth of China and India.
- Still, major *dispersion* persists across countries.

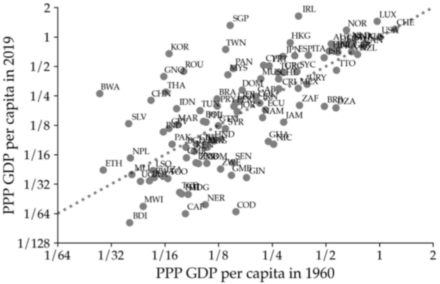

Figure 13.3: PPP GDP per Capita in 1960 and 2019 (U.S. = 1)

Figure: **2019 GDP per capita vs. 1960. Correlation** ≈ 0.75**, elasticity** ≈ 0.97**.**
No strong unconditional convergence, but no strong divergence either.

Motivation for Growth and Development Accounting

- Large differences in per capita incomes remain:
  - Congo vs. U.S. (factor of 64).
  - China $\approx 1/4$ and India $\approx 1/8$ of U.S. income in 2019.
- Why these differences?
- **Proximate causes:** physical capital, human capital, technology (TFP).
- **Deeper causes:** institutions, policies, geography, culture, etc.

Development Accounting: The Cobb-Douglas Setup

Consider a production function:

$$Y_{it} = K_{it}^{\alpha} \left(A_{it} H_{it} \right)^{1-\alpha}.$$

Dividing by L_{it} yields:

$$\frac{Y_{it}}{L_{it}} = \left(\frac{K_{it}}{Y_{it}} \right)^{\frac{\alpha}{1-\alpha}} \frac{H_{it}}{L_{it}} A_{it}.$$

- K/Y = physical capital-output ratio
- H/L = human capital per worker
- A = labor-augmenting TFP

Table 13.1: Growth Accounting for the U.S.

Period	Y/L	Contributions from K/Y	H/L	A
1948–2020	2.37	0.21	0.38	1.79
1948–1973	3.28	-0.18	0.27	3.19
1973–1995	1.54	0.46	0.36	0.72
1995–2007	2.80	0.32	0.40	2.08
2007–2020	1.64	0.43	0.59	0.63

Growth Accounting for the U.S. (Solow, 1957)

Table 13.1 shows decomposition:

$$\Delta \ln(Y/L) = \frac{\alpha}{1-\alpha}\Delta \ln(K/Y) + \Delta \ln(H/L) + \Delta \ln A.$$

- $\Delta \ln A$ (TFP growth) accounts for $\sim 75\%$ of growth over long periods.
- Capital deepening and human capital accumulation contribute the remaining $\sim 25\%$.
- TFP drives much of the *medium-run* shifts (e.g., post-1973 slowdown, mid-1990s productivity boom).

Table 13.2: Development Accounting in 2019

		Contributions from		
Statistic	Y/L	K/Y	H/L	A
Variance of log	1.00	0.14	0.08	0.57
Elasticity wrt Y/L		0.14	0.22	0.64
90/10 ratio	12.00	1.40	1.74	4.92

Development Accounting Across Countries

Table 13.2:

$$\ln\left(\frac{Y}{L}\right) = \frac{\alpha}{1-\alpha}\ln\left(\frac{K}{Y}\right) + \ln\left(\frac{H}{L}\right) + \ln(A).$$

- Capital intensity explains $\sim 12\%$ of income variation.
- Human capital explains $\sim 22\%$.
- TFP explains $\sim 66\%$.

The important thing is that TFP is the single largest contributor to cross-country income gaps.

Motivation for Multiple Sectors

- Empirical observation: **Relative price of investment** to consumption has trended down, especially in equipment.
- In poor countries, investment goods are relatively *more expensive* vs. consumption goods.
- This motivates a **two-sector** neoclassical model, with different technologies for producing consumption vs. investment goods. (Greenwood et al., 1997)

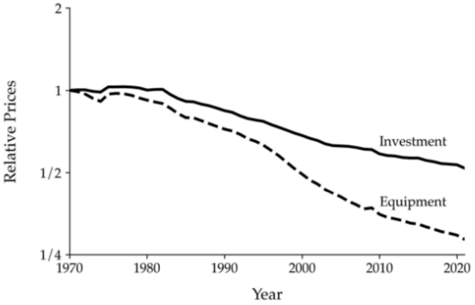

Figure 13.4: Relative price of equipment and investment in the U.S.

Figure: The data comes from the U.S. Bureau of Economic Analysis (NIPA Table 1.1.4). The series are relative to the price of consumption, and are each normalized to 1 in 1970. The price level for structures nearly doubled over 50 years by growing 1.3% per year relative to the consumption deflator. In contrast, the annual growth rate of the relative price of equipment and total investment were -2.47% and -1.26%, respectively.

Setup: Households

Representative household:

$$\max_{\{c(t),a(t)\}} \int_0^\infty e^{-(\rho-n)t} \frac{c(t)^{1-\sigma}-1}{1-\sigma}\, dt,$$

subject to:

$$\dot{a}(t) = [r(t) - n]\, a(t) + w(t)\, h - c(t),$$

with a no-Ponzi condition and n the population growth rate.

Setup: Firms

Production function (consumption numéraire):

$$Y(t) = K(t)^{\alpha}\left[A_y e^{\gamma_y t} L(t)\right]^{1-\alpha}.$$

- Output Y transforms one-for-one into consumption goods.
- Or transforms into $A_x e^{\gamma_x t}$ investment goods.
- γ_y = rate of *neutral* (Harrod) technical change.
- γ_x = rate of *investment-specific* technical change.

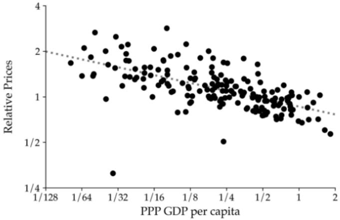

Figure 13.5: Price of investment relative to consumption in 2019 across countries

Figure: The data comes from the Penn World Table 10.0. The U.S. is normalized to 1 for both variables. The price of investment relative to consumption is calculated from the "pl i" and "pl c" variables. PPP GDP per capita is calculated as the ratio of the "rgdpo" and "pop" variables. The estimated elasticity is equal to -0.17 with a standard deviation of 0.02.

Equilibrium Conditions

Key Equations:

$$r(t) = \frac{R(t) - \delta P_x(t) + \dot{P}_x(t)}{P_x(t)},$$

$$K(t) = \frac{a(t)e^{nt}}{e^{\gamma_x t}/A_x}, \quad L(t) = e^{nt}h,$$

$$\text{Euler:} \quad \frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}\left(r(t) - \rho\right),$$

$$\text{Resource:} \quad \dot{K}(t) = Y(t)\frac{1}{P_x(t)} - \delta K(t).$$

Investment-specific technical change implies the **relative price** $P_x(t) = e^{-\gamma_x t}/A_x$ **declines** over time.

Balanced Growth Path (BGP)

- BGP definition: All variables grow at constant rates.
- Capital grows at $g_K = \frac{\gamma_x}{1-\alpha} + \gamma_y + n$.
- Consumption grows at $g_c = \alpha \frac{\gamma_x}{1-\alpha} + \gamma_y$.
- A constant fraction of income is saved in the long run, $\bar{s}$.
- **Result** : K/Y constant in *nominal* terms, but *real* (quality-adjusted) capital can still rise relative to real output if the price of investment is falling.

Comparisons with Data

- **Development Accounting (two-sector):**

$$\log\left(\frac{Y}{L}\right) = \frac{\alpha}{1-\alpha}\log\left(\frac{P_k K}{Y}\right) + \log\left(\frac{H}{L}\right) + \log(A).$$

- Investment-specific technology enters via P_k.
- Poor countries have higher P_k, leading to lower real capital intensity for the same nominal savings rate.
- TFP remains the biggest source of cross-country variation.

Simple AK Model

- Production: $Y = AK$.
- No diminishing returns, no role for labor.
- Household preferences yield:

$$\frac{\dot{c}(t)}{c(t)} = \frac{A - \delta - \rho}{\sigma}.$$

- **Purely factor-accumulation-based** growth: no TFP residual if all inputs are measured.
- Knife-edge condition $\alpha = 1$ is **critical**.
- Unlikely to match data if labor share $\approx 2/3$.

AK as a Reduced Form

- Could interpret K as *physical + human capital* with constant returns.
- Full-endogenous growth: invests in education each generation, sustaining growth in total human capital.
- Empirically, more schooling strongly correlates with higher level of income, but not necessarily **perpetually higher growth**.
- (Bils and Klenow, 2000): schooling changes typically show *level* rather than permanent growth effects.

Core Idea: Nonrivalry of Ideas

- (Romer, 1990) The stock of knowledge (varieties) N is nonrival.
- Final output:

$$Y = A\, L_y^{\phi} \int_0^N x(\nu)^{1-\phi}\, d\nu.$$

- Creating new variety ν requires R&D effort $\propto \frac{1}{\eta\, N(t)}$.
- Monopolistic competition: each variety is produced by its inventor at a markup.
- Knowledge spillover: $\dot{N}(t) = \eta\, N(t)\, L_r(t)$.

Equilibrium Growth in (Romer, 1990)

- With $\dot{N} = \eta \, N \, L_r$, and labor allocation $L = L_y + L_r$,

$$g_N \;=\; \frac{\dot{N}}{N} = \eta \, (L - L_y).$$

- Monopolistic pricing leads to markups but also **profits** that incentivize R&D.
- In the simplest version:

$$g^* = \frac{(1-\phi)\, \eta \, L - \rho}{1 - \phi + \sigma},$$

  (assuming no transition).
- **Strong scale effect:** bigger L leads to higher growth *rates*, contrary to cross-country data.

Semi-Endogenous Growth (Jones, 1995)

- If $\dot{N} = \eta N^{\epsilon} L_r$ with $\epsilon < 1$,
$$g_N \to n/(1-\epsilon),$$
- **Implication:** Long-run growth requires population growth n.
- Eliminates strong scale effects, but growth depends on ϵ and n.
- Policy can affect *levels* (human capital, capital) but not the **long-run** growth rate.

Motivation: Product Turnover

- Many innovations improve the *quality* of existing products rather than creating new varieties.
- *Creative destruction* (Schumpeter): new goods displace older, inferior ones.
- Empirical evidence: High rates of firm exit, job destruction, and job creation *within* narrowly defined industries (Haltiwanger et al.).

Firm exit rates (U.S., 1980–2022)

Creative destruction requires churn: small firms exit at double-digit annual rates; exit declines with size (.

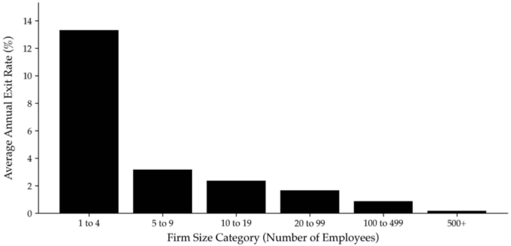

Figure 13.6: Firm exit rates in the U.S., 1980–2022

Job reallocation is large and persistent

Sum of creation & destruction $\approx$ 20–30%/year, trending down but still sizable ; Davis et al., 1998).

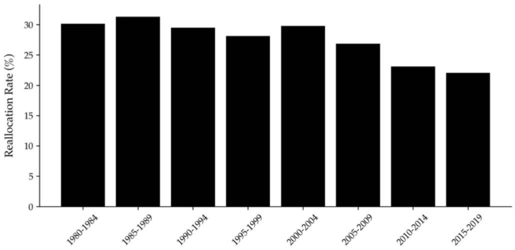

Figure 13.7: Job reallocation in the U.S., 1980–2019

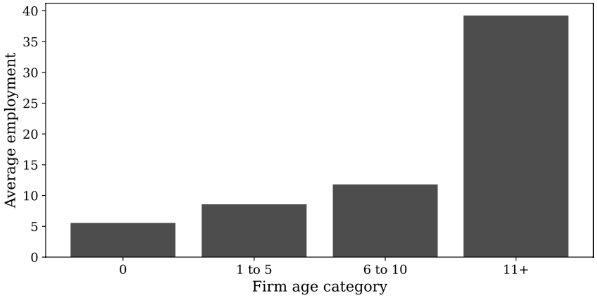

Figure 13.8: Firm size and firm age, 1988–2022

Source: U.S. Business Dynamics Statistics (BDS).

Entrant and exiter employment shares; firm size vs age

Entry share fell from $> 3\%$ to $\sim 2\%$; exiter share likewise fell. Young firms are small; selection and survivor growth matter (Hsieh and Klenow, 2014).

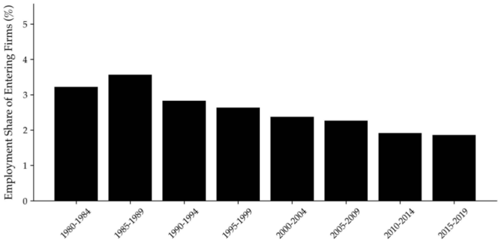

Figure 13.9: Entrant employment share in the U.S., 1980–2019

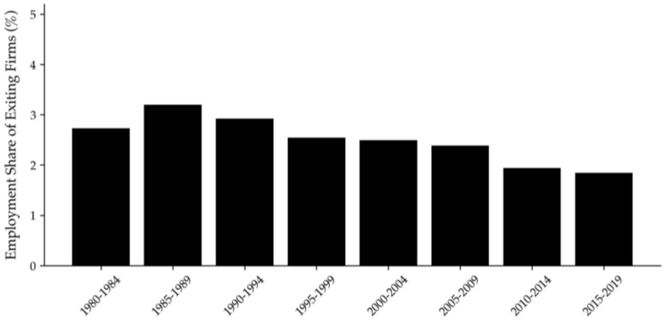

Figure 13.10: Exiting firm employment share in the U.S., 1980–2019

Industry exit and productivity growth

Sectors with higher exit grow faster on average ⇒ evidence consistent with creative destruction; see also Decker et al., 2016; Akcigit and Ates, 2023).

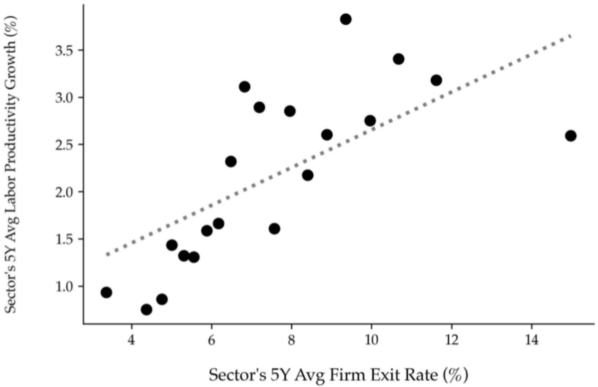

Figure 13.11: Growth and exit rates in the U.S., 1988–2022

Final-good technology and primitives

Continuum of intermediate varieties $\nu \in [0, 1]$ with quality $q(\nu, t)$. Final output:

$$Y(t) = \frac{A}{1-\phi} L(t)^{\phi} \int_0^1 q(\nu, t)\, x(\nu, t)^{\,1-\phi}\, \mathrm{d}\nu, \qquad 0 < \phi < 1,$$

where $x(\nu, t)$ is input of variety ν and $L(t) \equiv L$ is inelastic labor supply. This nests the CES across intermediates and yields diminishing returns to any single variety (Eq. (13.61)).

Intermediate demand & the wage (competitive final-good producers)

Final-good firms take $p(\nu, t)$ and $w(t)$ as given and maximize current profits:

$$\Pi(t) = \frac{A}{1-\phi} L^{\phi} \int q\, x^{1-\phi} \mathrm{d}\nu - \int p\, x\, \mathrm{d}\nu - wL.$$

FOCs imply

$$w(t) = \phi \frac{Y(t)}{L}, \qquad x(\nu, t) = \left[\frac{A\, q(\nu, t)}{p(\nu, t)}\right]^{1/\phi} L \quad \text{(quality-adjusted demand).}$$

Intermediate monopolists and "drastic" innovations

Intermediate producer for variety ν sets $p(\nu, t)$ taking $x(\nu, t)$ demand and marginal cost $\psi\, q(\nu, t)$ (lab-equipment). "Drastic" innovation ensures the leader need not fear lower-quality competitors if

$$\lambda \geq \left(\frac{1}{1-\phi}\right)^{\frac{1-\phi}{\phi}},$$

so the step size λ is large enough (Eq. (13.64) & App. 13.A.9). Optimal pricing:

$$p(\nu, t) = \frac{\psi}{1-\phi}\, q(\nu, t) \quad \Rightarrow \quad x(\nu, t) = \left(\frac{A(1-\phi)}{\psi}\right)^{\frac{1}{\phi}} L,$$

and variety profits

$$\pi(\nu, t) = \phi\, A \left(\frac{A(1-\phi)}{\psi}\right)^{\frac{1-\phi}{\phi}} q(\nu, t)\, L.$$

Aggregate output and average quality

Because $x(\nu, t)$ is constant across ν, aggregate output reduces to

$$Y(t) = \frac{A}{1-\phi}\left(\frac{A(1-\phi)}{\psi}\right)^{\frac{1-\phi}{\phi}} Q(t)\,L, \qquad Q(t) \equiv \int_0^1 q(\nu, t)\,\mathrm{d}\nu \quad \text{(average quality).}$$

Intermediates expenditure:

$$X(t) = \int_0^1 \psi\, q(\nu, t)\, x(\nu, t)\,\mathrm{d}\nu = \psi^{1-1/\phi}(A(1-\phi))^{1/\phi} Q(t)\,L.$$

R&D technology and quality dynamics

Research effort on line ν is $Z(\nu, t)$. Step innovations multiply quality by $\lambda > 1$:

$$q(\nu, t) = \lambda^{m(\nu,t)} q(\nu, 0),$$

Poisson arrival intensity:

$$z(\nu, t) = \eta \frac{Z(\nu, t)}{q(\nu, t)}, \qquad \eta > 0,$$

so higher quality makes further improvements harder (quality-adjusted effort). Aggregate research $Z(t) = \int_0^1 Z(\nu, t)\,\mathrm{d}\nu$. Aghion and Howitt, 1992)

Free entry and the Arrow replacement effect

With creative destruction, a successful entrant replaces the incumbent and captures its entire profit flow scaled up by λ. Free entry into research on each line imposes

$$\max_{Z(\nu,t)} \eta \frac{Z(\nu,t)}{q(\nu,t)} \cdot \lambda V(\nu,t) - Z(\nu,t) \quad \Rightarrow \quad \frac{\eta}{q(\nu,t)} \lambda V(\nu,t) = 1.$$

Incumbents would only gain the *increment* $(\lambda - 1)V$ if they self-cannibalize, which is strictly smaller $\Rightarrow$ **Arrow replacement effect**: entrants have stronger incentives than incumbents.

Value of a leading variety and household Euler

Leader value (discounting interest and destruction hazard):

$$V(\nu, t) = \int_t^{\infty} \exp\Big(- \int_t^s r(u)\,\mathrm{d}u \Big) \exp\Big(- \int_t^s z(\nu, u)\,\mathrm{d}u \Big)\, \pi(\nu, s)\,\mathrm{d}s.$$

Representative household (owns all firms) chooses $C(t)$ with CRRA utility and assets $A(t) = \int_0^1 V(\nu, t)\,\mathrm{d}\nu$:

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\sigma}.$$

BGP relationships (key three equations)

On a BGP, $g \equiv \dot{Y}/Y = \dot{C}/C = \dot{X}/X = \dot{Q}/Q$ and

$$\textbf{(i) Euler:} \quad r^* = \sigma g^* + \rho.$$

Free entry combined with V/q ratio gives

$$\textbf{(ii) FE:} \quad r^* + z^* = \underbrace{\lambda \eta \, \phi \, A \left(\frac{A(1-\phi)}{\psi} \right)^{\frac{1-\phi}{\phi}}}_{\equiv \Theta} L.$$

Quality growth equals step-size gain times arrival rate

$$\textbf{(iii) Quality law:} \quad g^* = (\lambda - 1)\, z^*.$$

Solving for g^*, r^*, z^*

Combine (i)–(iii). From (iii) $z^* = g^*/(\lambda - 1)$. Plug into (ii) and use (i):

$$\sigma g^* + \rho + \frac{g^*}{\lambda - 1} = \Theta L \quad \Rightarrow \quad \boxed{g^* = \frac{\Theta L - \rho}{\sigma + \frac{1}{\lambda - 1}}},$$

$$\boxed{r^* = \sigma g^* + \rho}, \qquad \boxed{z^* = \frac{g^*}{\lambda - 1}}.$$

Comparative statics: g^* rises in λ, η, A, L and falls in ρ, σ, ψ (intuitively: bigger steps, more efficient R&D, larger market scale $\Rightarrow$ higher growth).

Planner vs decentralized: growth and wedges

Planner internalizes (i) incremental social gain $(\lambda - 1)$ (no business stealing), (ii) monopoly markups (induces too little x), and (iii) knowledge externalities. Denote $\Theta_0 \equiv \eta\phi A\left(\frac{A(1-\phi)}{\psi}\right)^{\frac{1-\phi}{\phi}}$.
A compact way to write:

$$g^{SP} = \frac{(\lambda - 1)\Theta_0 L - \rho}{\sigma},$$

while g^{DE} uses $\Theta = \lambda\Theta_0$ and features the extra $1/(\lambda - 1)$ term from creative-destruction turnover. (Exact algebra & planner program: App. 13.A.10.)
Policy wedges: R&D subsidy, competition/markup policy, patent breadth/length.

Quality-ladder schematic

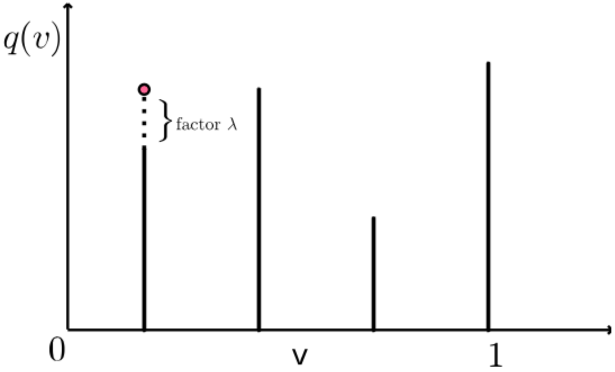

Figure 13.12: Quality ladders

Appendix: From monopolist FOC to $\pi(\nu, t)$

Given $x = \left[\frac{Aq}{p}\right]^{1/\phi} L$ and $MC = \psi q$,

$$\max_p \; p\,x - \psi q\, x \Rightarrow \; p = \frac{\psi}{1-\phi} q, \quad x = \left(\frac{A(1-\phi)}{\psi}\right)^{\frac{1}{\phi}} L,$$

$$\Rightarrow \; \pi(\nu, t) = (p - \psi q)x = \phi\, A \left(\frac{A(1-\phi)}{\psi}\right)^{\frac{1-\phi}{\phi}} q(\nu, t)\, L.$$

Deriving the BGP system

Free entry with $V/q = 1/(\lambda\eta)$ and $V/q = \dfrac{\pi/q}{r+z}$ gives

$$r^* + z^* = \lambda\eta \, \frac{\pi/q}{} = \lambda\eta\,\phi\,A\left(\frac{A(1-\phi)}{\psi}\right)^{\frac{1-\phi}{\phi}} L \equiv \Theta L.$$

Together with $r^* = \sigma g^* + \rho$ and $g^* = (\lambda - 1)z^*$ yields the closed forms on the preceding slide.

Empirical Observations on Creative Destruction

- Large **job reallocation** within sectors, consistent with product turnover.
- **Firm age and growth**: younger firms smaller on average but some grow quickly (e.g., tech start-ups).
- Over time, **entry rates** and **job reallocation** have declined in the U.S. Possibly linked to slowing TFP growth (Decker et al., 2016).

Policy and Welfare

- **Neoclassical model:** long-run growth is exogenous; policy mainly affects level and speed of convergence.
- **Endogenous growth:** policy can affect long-run growth via:
  - R&D subsidies (to internalize knowledge spillovers).
  - Intellectual property rights (balancing monopoly power vs. innovation incentives).
  - Education policies (raising long-run human capital).
  - Market structure and competition policy.
- Potential *trade-offs*: e.g. competition policy may reduce markups but also reduce R&D incentives.

Open Questions

- **Why has productivity growth slowed in many developed economies?**
- **What is the role of intangible investment, data, and AI?**
- **How do institutions and policies shape the adoption of new technologies?**
- Distributional impacts: *who* gains and who loses from growth and structural change?

Summary

- TFP **dominates** in explaining cross-country income *levels* and *growth over time*.
- Neoclassical models with investment-specific technical change match certain **price and capital** facts well.
- **Endogenous growth** frameworks emphasize *innovation*, *ideas*, *R&D*, and *market power*.
- **No single model** captures all empirical patterns perfectly, but each highlights critical mechanisms.
- Policy **can** matter if it affects innovation incentives, competition, and factor accumulation.

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Thank you!

Questions or comments?

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