Chapter 14: Real Business Cycles

Graduate Macroeconomics Slides

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Outline

What Are Business Cycles?

- Business cycles: Short-run fluctuations of aggregate economic activity around a long-run growth trend.
- Not necessarily literal, regular cycles like sine waves, but observable expansions and contractions.
- We focus on “real” business cycles (RBCs) — fluctuations largely driven by real shocks (e.g., technology), not just monetary factors.

Historical Overview

Early Contributions:

- **19th Century Crises**: Panic of 1825, recurring boom-bust episodes.
- **Schumpeter (1939)**: Emphasized technological innovation waves (Kitchin, Juglar, Kondratiev cycles).

Keynesian Revolution:

- (Keynes, 1936): Aggregate demand shortfalls and price/wage stickiness can drive persistent recessions.
- Widespread policy prescription for fiscal and monetary interventions.

1970s Stagflation:

- High inflation + high unemployment challenged Keynesian models.
- Rise of **Lucas critique** and emphasis on microfoundations (rational expectations).

From Lucas to RBC

- **Lucas and Rapping**: Intertemporal labor substitution; rational expectations.
- (Kydland and Prescott, 1982): Introduced a stochastic version of the neoclassical growth model (the RBC model).
- RBC model: business cycle fluctuations primarily driven by TFP (technology) shocks.
- Methodological innovation: solving models numerically and comparing simulated data to empirical data.

U.S. Time-Series Data on GDP

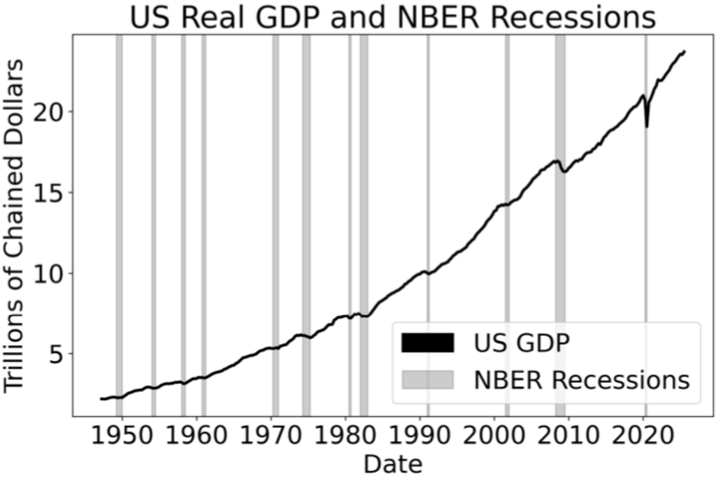

Figure 14.1: U.S. Real GDP and NBER Recessions

U.S. Time-Series Data on GDP

Key Observations:

- Strong upward trend in real GDP since WWII.
- The **NBER** identifies recessions (peaks, troughs) via a committee, but does so retroactively.
- To study business cycles, we decompose time series into *trend* + *cyclical* components.

Common Detrending Techniques:

- **Hodrick-Prescott filter** (HP)
- **Band-pass filter**
- First-differences

NBER Recessions

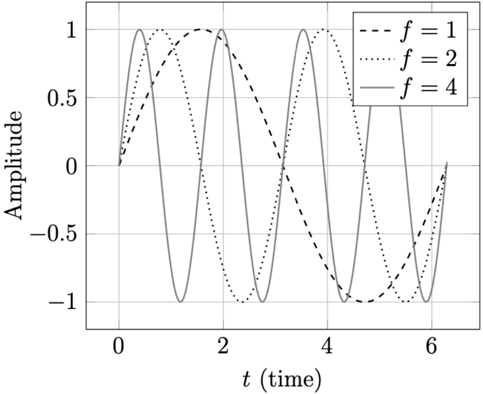

Figure 14.2: Sine waves with different frequencies

Filters

- Business cycle analysis requires separating long-run trends from short-run fluctuations.
- Raw macroeconomic data (e.g., GDP) contains both components.
- Goal: transform data into a stationary series suitable for studying cyclical dynamics.
- Example: Actual vs. Trend Log of U.S. GDP (HP filter).

Underlying Data Generating Processes (DGPs)

- Time series can be decomposed as:

$$Y_t = \tau_t + Y_{c,t}$$

  - τ_t: trend component
  - $Y_{c,t}$: cyclical component
- Trend-stationary assumption: $\tau_t = y_0 e^{\gamma t}$, cycles are i.i.d. shocks.
- Difference-stationary assumption: $\Delta Y_t = \bar{Y} + \phi(L)\epsilon_t$.
- Both assumptions can approximate finite time series reasonably well.

The Hodrick-Prescott (HP) Filter

- Most widely used filter in business cycle analysis.
- Decomposes series into:

$$Y_t = \tau_t + Y_{c,t}$$

- Trend τ_t obtained by solving:

$$\min_{\{\tau_t\}} \sum_{t=1}^{T} (Y_t - \tau_t)^2$$

subject to smoothness constraint:

$$\sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \leq \mu$$

HP Filter: Lagrangian Formulation

- Using Lagrange multiplier λ:

$$\min_{\{\tau_t\}} \left[\sum_{t=1}^{T} (Y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} ((\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1}))^2 \right]$$

- λ controls smoothness of the trend:
  - $\lambda = 0$: linear trend
  - Large λ: smoother trend, closer to linear
- Conventional choices:
  - Quarterly data: $\lambda = 1600$
  - Annual data: $\lambda = 6.25$

Matrix Solution of HP Filter

- First-order conditions yield linear system:

$$\tau - \lambda(\Delta\tau_t - 2\Delta\tau_t + \Delta\tau_{t+2}) = Y_t$$

- Equivalent to solving:

$$(I + \lambda A)\tau = Y$$

where I is identity, A is banded matrix capturing second differences.

- Solution:

$$\tau = (I + \lambda A)^{-1} Y$$

Stylized Business-Cycle Facts (U.S.)

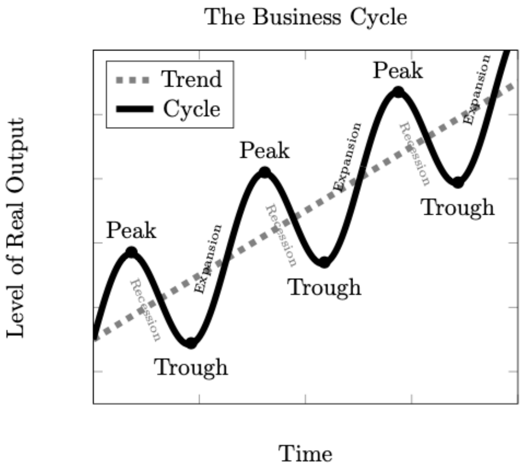

Figure 14.3: Stylized representation of business cycles

Stylized Business-Cycle Facts (U.S.)

- **Consumption** is less volatile than output; **investment** is more volatile.
- **Hours worked** and **employment** are procyclical and about as volatile as output.
- **Real wages** and **price level** tend to be less volatile.
- **TFP** strongly correlates with output (procyclical) but correlations can change over subperiods.
- **Unemployment** is strongly countercyclical.

Persistence:

- Output growth and TFP exhibit significant serial correlation.
- ”Great Moderation” after mid-1980s: reduced output volatility.

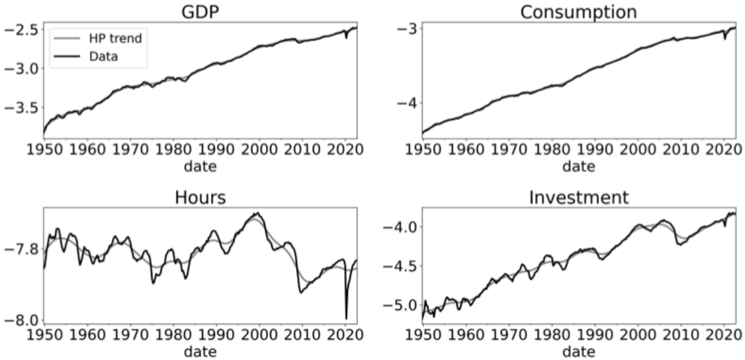

Figure 14.5: Actual and Trend of Logs of U.S. Aggregates

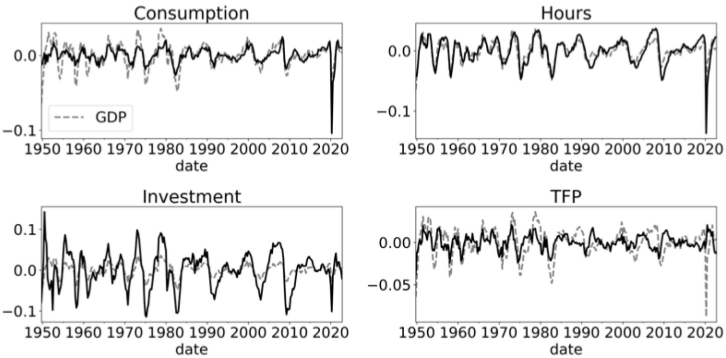

Figure 14.6: Deviations from Trend of U.S. Aggregates

Table 14.1: Business cycle moments U.S. Data 1949-2022

Variable x	Standard Deviation (%)	Relative Std to σ_y	Auto-correlation	Cross correlation of x with $y(t-1)$	$y(t)$	$y(t+1)$
Output (y)	1.63	1.00	0.78	0.78	1.00	0.78
Consumption	1.07	0.65	0.64	0.58	0.75	0.53
Gov. Consumption	3.02	1.85	0.89	0.23	0.15	0.04
Investment	4.37	2.68	0.86	0.61	0.77	0.71
Employment	1.58	0.97	0.81	0.79	0.79	0.48
Hours	2.11	1.29	0.80	0.77	0.86	0.61
Unemployment	15.63	9.57	0.80	-0.79	-0.83	-0.55
Lab. Productivity	1.15	0.70	0.72	-0.03	0.27	0.38
Wages	1.24	0.76	0.72	-0.03	-0.00	0.15
Price Level	0.92	0.56	0.92	-0.02	-0.10	-0.21
TFP	0.90	0.55	0.75	0.17	0.51	0.52
Fed Funds Rate	3.60	2.20	0.97	0.23	0.18	0.08

Table 14.2: Business cycle moments U.S. Data 1985-2022

Variable x	Standard Deviation (%)	Relative Std to σ_y	Auto-correlation	Cross correlation of x with $y(t-1)$	$y(t)$	$y(t+1)$
Output (y)	1.24	1.00	0.64	0.64	1.00	0.64
Consumption	1.20	0.97	0.56	0.56	0.86	0.43
Gov. Consumption	1.26	1.02	0.83	-0.32	-0.38	-0.39
Investment	3.64	2.94	0.90	0.63	0.79	0.70
Employment	1.56	1.26	0.72	0.69	0.78	0.34
Hours	2.19	1.77	0.74	0.70	0.88	0.52
Unemployment	15.60	12.59	0.74	-0.72	-0.83	-0.43
Lab. Productivity	1.09	0.88	0.76	-0.47	-0.29	-0.05
Wages	1.46	1.18	0.69	-0.21	-0.31	-0.06
Price Level	0.68	0.55	0.93	0.37	0.30	0.14
TFP	0.73	0.59	0.78	-0.33	-0.03	0.10
Fed Funds Rate	2.83	2.29	0.99	0.30	0.29	0.25

Data Over Different Subsamples

- **Pre-1985 vs. Post-1985:**
  - Output volatility significantly declined after 1985 (Great Moderation).
  - **Labor productivity and wages** changed cyclical patterns, becoming less procyclical.
  - Government consumption became more *counter*cyclical, suggesting active stabilization policies.
- **Implication:** RBC models calibrated to earlier data might overestimate how tightly labor productivity and output move together after 1985.

Beyond Unconditional Moments

Impulse Response Functions (IRFs):

- Trace out dynamic responses of variables to structural shocks.
- Estimated via:
  - **Vector Autoregressions (VARs)** + identification (timing, sign restrictions, etc.).
  - **Local Projections (LPs)** using identified structural shocks directly in regressions.
- **Typical Finding:** TFP shocks do correlate with output and investment, consistent with RBC ideas.

Table 14.3: Business cycle moments U.S. Data 1949-1984

Variable x	Standard Deviation (%)	Relative Std to σ_y	Auto-correlation	Cross correlation of x with $y(t-1)$	$y(t)$	$y(t+1)$
Output (y)	1.97	1.00	0.84	0.84	1.00	0.84
Consumption	0.89	0.45	0.81	0.66	0.75	0.69
Gov. Consumption	4.15	2.11	0.90	0.35	0.26	0.12
Investment	5.00	2.54	0.83	0.59	0.76	0.70
Employment	1.56	0.79	0.90	0.89	0.83	0.59
Hours	2.00	1.02	0.88	0.88	0.90	0.70
Unemployment	15.66	7.96	0.87	-0.89	-0.88	-0.66
Lab. Productivity	1.21	0.61	0.68	0.23	0.63	0.66
Wages	0.93	0.47	0.77	0.13	0.33	0.40
Price Level	1.14	0.58	0.91	-0.17	-0.28	-0.37
TFP	1.05	0.54	0.73	0.42	0.76	0.73
Fed Funds Rate	3.89	1.98	0.95	0.25	0.16	0.01

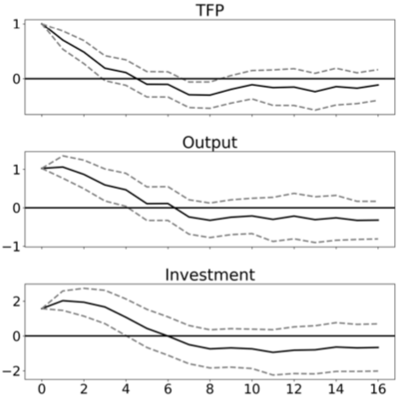

Figure 14.7: IRF to TFP

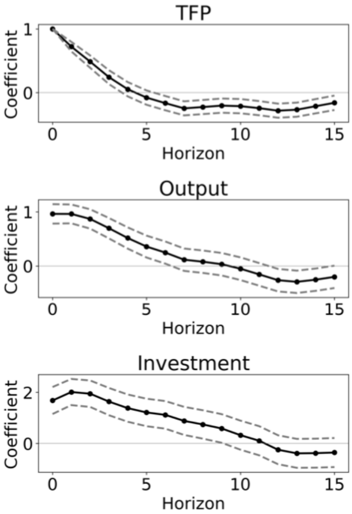

Figure 14.8: IRF to TFP

Core Setup

Representative Agent:

- Infinite-horizon, discount factor $\beta \in (0, 1)$.
- Utility from consumption c and leisure l:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t).$$

Technology:

- Cobb-Douglas (often): $Y_t = z_t F(K_t, N_t)$, with $N_t = 1 - l_t$.
- z_t is TFP, usually follows an AR(1): $\ln z_t = \rho \ln z_{t-1} + \sigma \varepsilon_t$.
- Capital evolves: $K_{t+1} = (1 - \delta) K_t + I_t$.

Social Planner Problem

A simpler way (due to efficiency of competitive equilibrium) is to solve:

$$\max_{\{c_t, n_t, K_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - n_t)$$

subject to

$$c_t + I_t = z_t F(K_t, n_t), \quad K_{t+1} = (1 - \delta) K_t + I_t.$$

Key FOCs:

$$\text{(Intratemporal)}: \; U_2(c_t, 1 - n_t) = z_t F_n(K_t, n_t)\, U_1(c_t, 1 - n_t),$$

$$\text{(Intertemporal)}: \; U_1(c_t, 1 - n_t) = \beta\, \mathbb{E}\Big[\big(z_{t+1} F_K(K_{t+1}, n_{t+1}) + 1 - \delta\big) U_1(c_{t+1}, 1 - n_{t+1})\Big].$$

Mechanics of RBC Fluctuations

Positive TFP Shock $\Rightarrow$

- **Higher marginal product of labor:** Households increase labor supply if intertemporal substitution effect is strong.
- **Higher expected returns on capital:** Encourages more investment.
- Consumption smoothing moderates how much immediate consumption rises.
- End result: *Amplification* from hours and capital investment.

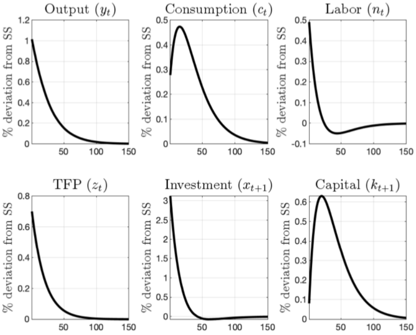

Figure 14.9: TFP Shock in RBC Model

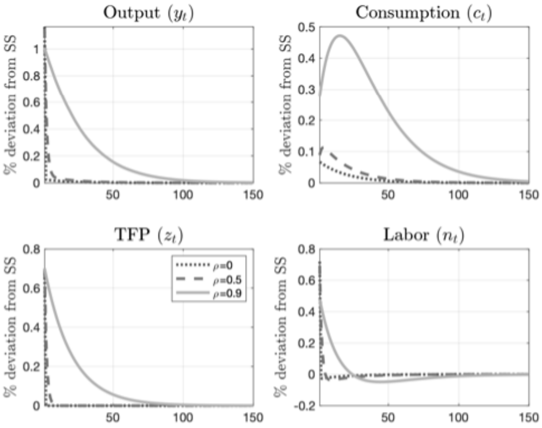

Figure 14.10: Sensitivity to TFP Shock Persistence in RBC Model

Calibration à la (Kydland and Prescott, 1982)

Typical parameter choices:

- $\beta \approx 0.99$ (quarterly).
- $\delta \approx 0.025$ (quarterly).
- α in $F(K, N) = K^{\alpha} N^{1-\alpha}$ is around 0.33–0.36.
- ρ of TFP ≈ 0.95, $\sigma_{\varepsilon} \approx 0.007$.
- Preferences must respect balanced growth: often CRRA in c, sometimes log in c and $\log(1 - n)$ form.

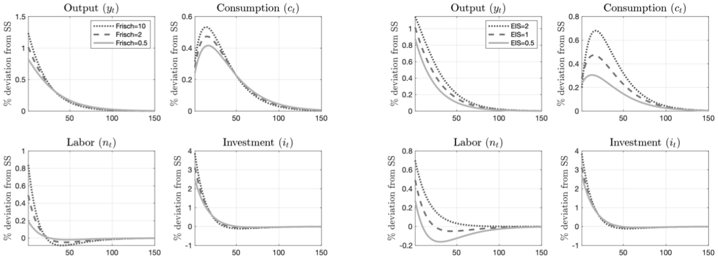

Figure 14.11: Sensitivity to EIS and Frisch Elasticity in RBC Model

Key RBC predictions:

- Investment more volatile than output; consumption less volatile than output.
- Procyclical labor, procyclical TFP and real wage (in many calibrations).
- RBC explains a sizable fraction of output fluctuations via TFP shocks.

Limitations of the Baseline RBC Model

- **Labor volatility**: The basic RBC model often underpredicts how much hours fluctuate relative to output, unless the Frisch elasticity is set quite high.
- **Strong co-movements**: One-shock model can produce overly high correlations among all real variables.
- **Changing empirical relationships**: Post-1985 data reveal different labor-productivity correlations.
- **No explicit unemployment**: Basic model has no involuntary unemployment or search frictions.

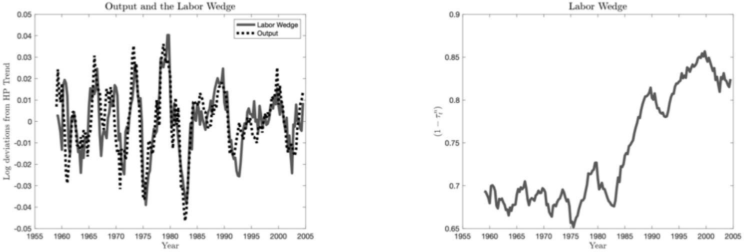

Figure 14.12: The estimated labor wedge from BCA

Indivisible Labor and Employment

- (Hansen, 1985), (Rogerson, 1988): Introduce *indivisible labor* and lotteries.
- Households either work full-time or not at all; fraction of workers adjusts at the extensive margin.
- Reconciles small intensive-margin labor elasticity in micro data with large extensive-margin elasticity in aggregate data.
- Generates more volatility in employment, closer to observed data.

Capital Adjustment Costs

- (Hayashi, 1982): Firms face convex costs when adjusting capital stock.
  $K_{t+1} = (1-\delta)K_t + I_t - \Psi(I_t, K_t)$.
- Smooths out investment spikes and can generate more persistence in response to TFP shocks.
- Tobin's q: the shadow price of installed capital, moves with adjustments.
- Widely used in RBC and New Keynesian DSGE models.

Variable Capacity Utilization

- (Greenwood et al., 1988): Firms choose how intensively to utilize capital, u_t.
- Production: $Y_t = z_t F(u_t K_t, N_t)$.
- Depreciation depends on utilization: $K_{t+1} = (1 - \delta(u_t))K_t + I_t$.
- Another margin of cyclical adjustment (e.g., running extra shifts).
- Increases amplification of TFP shocks (more short-run capital services).

Additional Shock Processes

- **Investment-Specific Technology Shocks:** (Greenwood et al., 1997, 2000)
  $K_{t+1} = (1-\delta)K_t + q_t I_t, \quad q_t$stochastic.
  - Distinguish consumption-good technology from investment-good technology.
  - Explains relative price movements of investment, capital-embodied technical change.
- **Government Spending Shocks** $C_t + I_t + G_t = Y_t$.
  - Fiscal policy can be procyclical or countercyclical.
  - Helps break too-strong correlation of wages and hours in baseline RBC.

Business Cycle Accounting (BCA)

(Chari et al., 2007) propose:

- Decompose data into four "wedges" that distort RBC equations:
  1. **Efficiency wedge** (z_t),
  2. **Labor wedge** ($1 - \tau_t^n$),
  3. **Investment wedge** ($1/(1 + \tau_t^i)$),
  4. **Government wedge** (G_t).
- Estimate these wedges from the data, examine which wedges matter most.
- **Finding**: Efficiency and labor wedges explain bulk of cyclical fluctuations.

The Labor Wedge and "Jobless Recoveries"

- Labor wedge is strongly correlated with output; in downturns, wedge spikes ("tax" on work).
- Could represent frictions like search/matching, unemployment benefits, or other labor market distortions.
- **Recent theories**: Variation in unemployment insurance or slower re-hiring rates can drive "jobless recoveries."
- Alternatively, long-run changes in female labor participation and composition effects explain shifts in aggregate labor patterns.

Successes of RBC Models

- Provided a **microfounded** framework for macro: preferences, technology, equilibrium.
- Quantitative discipline via **calibration** and direct confrontation with data.
- Explained key stylized facts: procyclicality of output, consumption, investment, hours; relative volatilities.
- Set the stage for **modern DSGE** and New Keynesian frameworks.

Critiques and Ongoing Debates

- **Labor market** frictionless assumption often at odds with micro-evidence on unemployment, wage rigidity.
- Strong reliance on a single shock (TFP) for short-run fluctuations may not suffice, especially post-1985 era.
- High **Frisch elasticities** needed to match hours data vs. smaller micro estimates.
- Insufficient **endogenous persistence** if TFP shock is not highly autocorrelated.

Where Has the Literature Gone?

- **Real-Financial Linkages**: RBC + credit frictions, incomplete markets.
- **Heterogeneous Agent (HA) Models**: Households face borrowing constraints, idiosyncratic risk.
- **New Keynesian RBC**: Sticky prices, monetary policy, but RBC core (dynamic general equilibrium) still fundamental.
- **Empirical Tools**: VARs, Local Projections, microdata-based identification of structural shocks.
- **Policy Analysis**: RBC logic extended to fiscal policy, structural reforms, labor market policies, etc.

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Thank you!

Questions or comments?

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