Chapter 16: Asset Prices

Graduate Macroeconomics Slides

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Outline

Why many assets?

- One-asset growth models deliver a single return; real economies feature many assets and returns.
- Households choose *both* total saving and portfolio shares; price changes reallocate wealth across heterogeneous portfolios.
- Complete vs. incomplete markets: with completeness, returns discipline preferences/beliefs; with incompleteness, market structure affects allocations and welfare.

Key notation used throughout

- Prices $p_t \in \mathbb{R}^N$, dividends $d_t \in \mathbb{R}^N$, holdings $\theta_{i,t}$, consumption $c_{i,t}$.
- Gross return $R^n_{t+1} = (d^n_{t+1} + p^n_{t+1})/p^n_t$, risk-free R^f_t.
- State price density / SDF M_{t+1} with pricing equation $\mathbb{E}_t[M_{t+1} R^n_{t+1}] = 1$.
- Expectations $\mathbb{E}_t[\cdot]$, variance $\mathrm{Var}(\cdot)$, covariance $\mathrm{Cov}(\cdot, \cdot)$.

Participation and heterogeneity

- Homeownership widespread; equity participation varies by country and cohort.
- Young often levered into housing; older cohorts hold both housing and equity.
- Risk comes from cash flows (dividends/rents) and resale prices.

Takeaway

Portfolio composition and participation differ systematically across groups, shaping how price changes redistribute wealth.

Price-cash-flow ratios

- Track P/D for equities and P/Rent for housing to net out common growth.
- These ratios display large swings; sometimes comoving, sometimes diverging.
- Question: Do movements reflect cash-flow news or discount-rate news?

Evidence: ratio dynamics

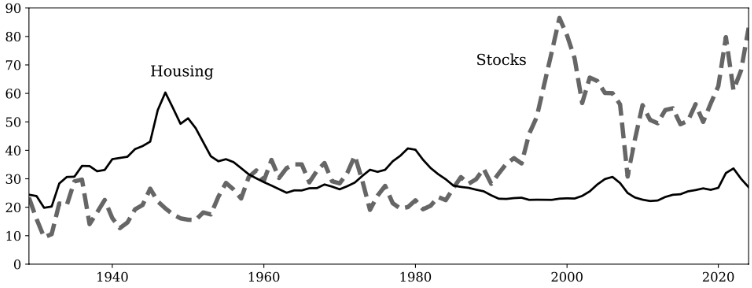

Figure 16.1: The ratio of asset value to cash flow for housing and stocks in the United States, 1930-2023.

Environment and feasibility

Let ω_t denote a date-t history with probability $\pi_t(\omega_t)$. Agent i receives $y_{i,t}(\omega_t)$ and chooses $c_{i,t}(\omega_t)$.

$$\sum_i c_{i,t}(\omega_t) \leq \sum_i y_{i,t}(\omega_t) \equiv y_t(\omega_t).$$

Preferences (CRRA or Epstein–Zin):

$$U_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, u(c_{i,t}), \quad u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

Epstein–Zin separates risk aversion γ from EIS $1/\psi$ when the data require high γ but reasonable EIS.

Planner, Arrow–Debreu prices, First Welfare Theorem

Planner problem:

$$\max_{\{c_i\}} \sum_i \lambda_i U_i(c_i) \quad \text{s.t.} \quad \sum_i c_{i,t}(\omega_t) \leq y_t(\omega_t).$$

FOC: $\lambda_i \, \partial U_i / \partial c_{i,t}(\omega_t) = \mu_t(\omega_t)$.

Arrow–Debreu equilibrium with state prices $p_t^0(\omega_t)$:

$$\sum_{t,\omega_t} p_t^0(\omega_t)\, c_{i,t}(\omega_t) \leq \sum_{t,\omega_t} p_t^0(\omega_t)\, y_{i,t}(\omega_t),$$

and $\partial U_i / \partial c_{i,t}(\omega_t) = \alpha_i p_t^0(\omega_t)$. Setting $\lambda_i = 1/\alpha_i$ reproduces the planner allocation under complete markets .

Sequential trading, completeness, and aggregation

Let $p_t \in \mathbb{R}^N$ be ex-dividend prices and $d_t \in \mathbb{R}^N$ dividends. With holdings $\theta_{i,t}$:

$$c_{i,t} + p_t^\top \theta_{i,t} \le y_{i,t} + (d_t + p_t)^\top \theta_{i,t-1}.$$

Markets are complete when any AD payoff is spanned by trading $\{\theta_{i,t}\}$. Under completeness, sequential and AD equilibria coincide and a representative-agent SDF prices all assets.

Law of One Price and FTAP

Date-0 prices $p \in \mathbb{R}^N$, date-1 payoffs $D \in \mathbb{R}^{S \times N}$. If strictly positive state prices $q \in \mathbb{R}^S_{++}$ exist:

$$p = D^\top q.$$

Fundamental Theorem of Asset Pricing

No arbitrage $\Leftrightarrow$ there exist strictly positive state prices q with $p = D^\top q$. If $\mathrm{rank}(D) = S$, markets are complete and q is unique.

Geometric illustration

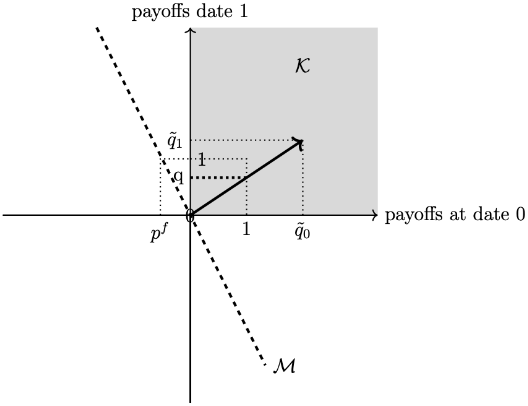

Figure 16.2: The set of attainable payoffs $\mathcal{M}$ and the set of arbitrages $\mathcal{K}$ separated by a linear function $F(x) = \tilde{q}_0 x_0 + \tilde{q}_1^\top x_1 = 0$ for $x \in \mathcal{M}$ with coefficients $\tilde{q}$ that are orthogonal to the elements of $\mathcal{M}$.

SDF, risk-neutral probabilities, risk premia

Subjective state prices:

$$q_i(\omega) = \frac{\partial U_i / \partial c_1(\omega)}{\partial U_i / \partial c_0}.$$

Define SDF $M(\omega) = q(\omega)/\pi(\omega)$ with objective probabilities $\pi(\omega)$. For asset n:

$$1 = \mathbb{E}\big[M(\omega) R_n(\omega)\big].$$

Risk-neutral probabilities $\pi^*(\omega) = q(\omega)/\sum_{\omega'} q(\omega')$; $p_n = R_f^{-1}\mathbb{E}^*[d_n]$.

$$\mathbb{E}[R_n] - R_f = -\frac{\mathrm{Cov}(R_n, M)}{\mathbb{E}[M]} \quad \Rightarrow \quad \text{only SDF-covarying risk is priced.}$$

Hansen–Jagannathan bound

Sharpe ratio $S_n = (\mathbb{E}R_n - R_f)/\sqrt{\text{Var}(R_n)}$. By Cauchy–Schwarz,

$$|S_n| \leq \frac{\sqrt{\text{Var}(M)}}{\mathbb{E}[M]}.$$

Takeaway

The maximum Sharpe ratio in the data implies a lower bound on $\sqrt{\text{Var}(M)}/\mathbb{E}[M]$ that any model SDF must satisfy.

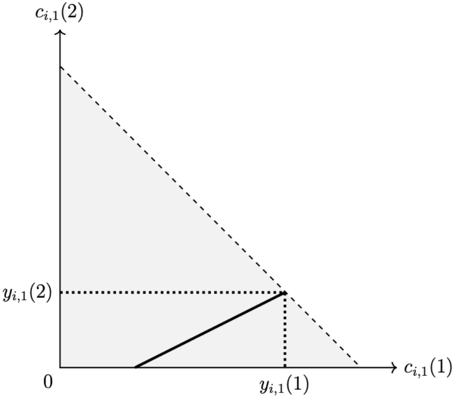

Figure 16.3: Budget sets with complete markets as dashed line and with only a riskless bond as solid line

Euler equations and premium decomposition

Present value:

$$p_t^n = \mathbb{E}_t\big[M_{t+1}(d_{t+1}^n + p_{t+1}^n)\big], \quad R_{t+1}^n = \frac{d_{t+1}^n + p_{t+1}^n}{p_t^n}.$$

Euler:

$$\mathbb{E}_t[M_{t+1}R_{t+1}^n] = 1, \qquad R_t^f = \mathbb{E}_t[M_{t+1}]^{-1}.$$

Subtracting the risk-free:

$$\mathbb{E}_t[R_{t+1}^n] - R_t^f = -\frac{\text{Cov}_t(M_{t+1}, R_{t+1}^n)}{\mathbb{E}_t[M_{t+1}]}.$$

Consumption-based SDFs

CRRA expected utility:

$$M_{t+1}^{EU} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}.$$

Epstein–Zin:

$$M_{t+1}^{EZ} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-1/\psi} R_{w,t+1}^{\theta-1}, \qquad \theta = \frac{1-\gamma}{1-1/\psi}.$$

Takeaway

Epstein–Zin decouples risk aversion γ and EIS $1/\psi$, improving quantitative fit.

Campbell–Shiller log-linear identity

Log price p_t, log dividend d_t:

$$p_t - d_t \approx \kappa + \rho(p_{t+1} - d_{t+1}) + \Delta d_{t+1} - r_{t+1}.$$

Forward iteration:

$$p_t - d_t \approx \frac{\kappa}{1-\rho} + \sum_{j=1}^{\infty} \rho^{j-1}\left(\Delta d_{t+j} - r_{t+j}\right).$$

High D/P today forecasts low future dividend growth or high future returns; empirically, most variation forecasts returns.

Lucas tree with Markov consumption growth

Consumption $C_{t+1} = C_t G_{t+1}$, $G_{t+1} \in \{G_L, G_H\}$ with transition matrix $\begin{pmatrix} \phi & 1-\phi \\ 1-\phi & \phi \end{pmatrix}$. CRRA SDF $M_{t+1} = \beta G_{t+1}^{-\gamma}$.

$$v_t \equiv \frac{P_t}{C_t} = \mathbb{E}_t\left[\beta G_{t+1}^{1-\gamma}(1 + v_{t+1})\right].$$

Vector form: $\boldsymbol{v} = \beta A(1 + \boldsymbol{v})$ with $A_{ij} = \Pi_{ij} G_j^{1-\gamma}$.

$$R^s_{ij} = G_j \frac{1 + v_j}{v_i}, \qquad R^f_i = \left(\sum_j \Pi_{ij} \beta G_j^{-\gamma}\right)^{-1}.$$

Calibrated to postwar data, the model yields too small an equity premium or too high a safe rate (Mehra and Prescott, 1985).

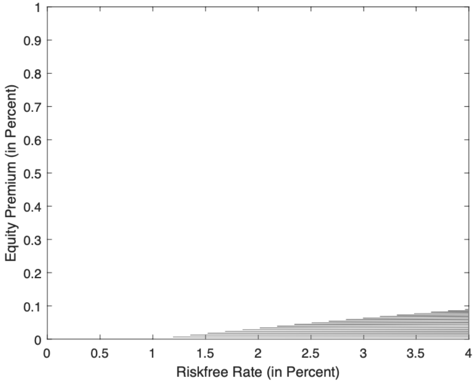

Figure 16.4: The shaded area shows the equity premium for equilibria in which the riskfree rate is below 4 percent for preference parameters $\beta \in [0, 1]$ and $\gamma \in [0, 100]$.

Closed forms for r^f and the equity premium

Assume $\log G_{t+1} \sim \mathcal{N}(\mu_g, \sigma_g^2)$ and CRRA. Then $\log M_{t+1} = \log \beta - \gamma \log G_{t+1}$ and

$$r_t^f \equiv \log R_t^f = -\log \beta + \gamma \mu_g - \tfrac{1}{2}\gamma^2 \sigma_g^2.$$

Let r_{t+1}^s be log equity return. CCAPM implies

$$\mathbb{E}_t(r_{t+1}^s) - r_t^f + \tfrac{1}{2}\sigma_r^2 = \gamma \ \mathrm{Cov}_t\big(\log G_{t+1},\ r_{t+1}^s\big).$$

Because $\mathrm{Cov}(\log G, r^s)$ is small, matching the premium requires implausibly high γ, which misfires on r^f.

Excess volatility under i.i.d. growth

Under CRRA with i.i.d. lognormal growth,

$$v_t = \beta\,\mathbb{E}\left[G^{1-\gamma}\right] + \beta^2\,\mathbb{E}\left[G^{1-\gamma}\right]^2 + \cdots = \bar{v},$$

and

$$r^s_{t+1} = \log G_{t+1} + \log\left(\frac{1+\bar{v}}{\bar{v}}\right).$$

Hence $\mathrm{Var}(r^s) \approx \mathrm{Var}(\log G)$, far below observed return volatility (excess volatility puzzle).

Dividend yields and return predictability

Campbell–Shiller: movements in D/P reflect dividend-growth news or discount-rate news. Empirically, high D/P predicts high subsequent excess returns at medium/long horizons, but weakly predicts dividend growth

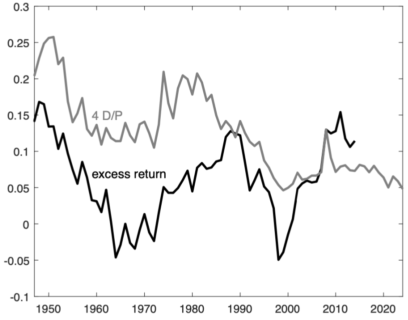

Figure 16.5: Dividend-price ratio together with excess returns on stocks over the next 10 years.

Table 2: Return forecasting regressions

Horizon k	b	$t(b)$	R^2
1 year	2.41	1.78	0.04
5 year	16.85	2.03	0.17
10 year	51.49	3.20	0.28

Note: The regression equation is $r^s_{t\to t+k} - r^f_{t\to t+k} = a + bd_t/p^s_t + \varepsilon_{t+k}$. The dependent variable is the excess return on the S&P 500 over the 3-month T-bill rate. Data are annual 1947-2024. The k-year regression t-statistic uses the Hansen–Hodrick (1980) correction when $k > 1$.

Diagnostics: variance bounds, spectra, HJ distance

- **Variance bounds:** compare price volatility to discounted dividend volatility; large gaps point to discount-rate variation or misspecified cash flows.
- **Spectral tools:** allocate variation across horizons; premia move at medium and long horizons.
- **HJ distance:** measures how far a candidate SDF is from the set that prices a chosen cross-section of returns.

Orientation: Epstein–Zin and long-run risks

- **Epstein–Zin** decouples risk aversion and EIS, easing premium vs. safe-rate tension.
- **Long-run risks:** small persistent growth and time-varying volatility increase $\text{Cov}(R, M)$ without excessive consumption volatility.

Summary

- No-arbitrage $\Leftrightarrow$ strictly positive state prices $\Leftrightarrow$ an SDF that prices all assets.
- Only SDF-covarying risk is priced; smooth consumption makes this covariance too small under CRRA.
- Under i.i.d. growth, P/D is constant and cannot match equity volatility.
- Dividend yields forecast returns, indicating time-varying discount rates.
- Epstein–Zin and long-run risks reconcile evidence while preserving Euler discipline.

References I

Mehra, R. and Prescott, E. C. (1985). The equity premium: A puzzle. *Journal of Monetary Economics*, 15(2):145–161.

Thank you!

Questions or comments?

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