

# Chapter 20: Frictional Labor Markets

## Graduate Macroeconomics Slides

Orhan Torul

Boğaziçi University

January 23, 2026



# Outline

- Introduction
- Some Labor Market Facts
- A Simple Model of Unemployment
- The Diamond-Mortensen-Pissarides (DMP) Model
- Labor Market Facts, Once Again
- The Unemployment Volatility Puzzle
- Endogenous Separation
- Frictions in the Neoclassical Growth Model
- Heterogeneity & Frictional Dispersion
- Conclusion



# Motivation: RBC Models and Labor Markets

- Early real business cycle (RBC) models assume frictionless labor markets.
- In a frictionless setting, wages instantly adjust to equate labor supply and labor demand.
- Unemployment is effectively ignored in this frictionless paradigm.
- However, real-world data shows unemployment is significant and procyclical in recessions.



# Why Study Unemployment?

- The unemployment rate is a key indicator of economic health.
- Social costs of unemployment are large, especially during recessions.
- Government policies like unemployment insurance and job training affect labor market outcomes.
- We need a formal model with endogenous unemployment to analyze these policies.



# Introducing Labor Market Frictions

- Wage rigidity can cause unemployment if wage is above market-clearing level.
- Minimum wages, unions, or efficiency wages may keep wages high.
- Another channel: Search frictions – it takes time and effort for workers and firms to find each other.
- This chapter focuses on search frictions and matching models, especially the DMP (Diamond-Mortensen-Pissarides) framework.



# U.S. Unemployment Rate (Postwar Period)

- **Figure 20.1:** Historical unemployment rate.
- Shaded regions indicate recessions; unemployment rises in recessions.
- No clear long-run trend, but cyclical variability is strong.
- Notable spikes in Great Recession and COVID-19 period.

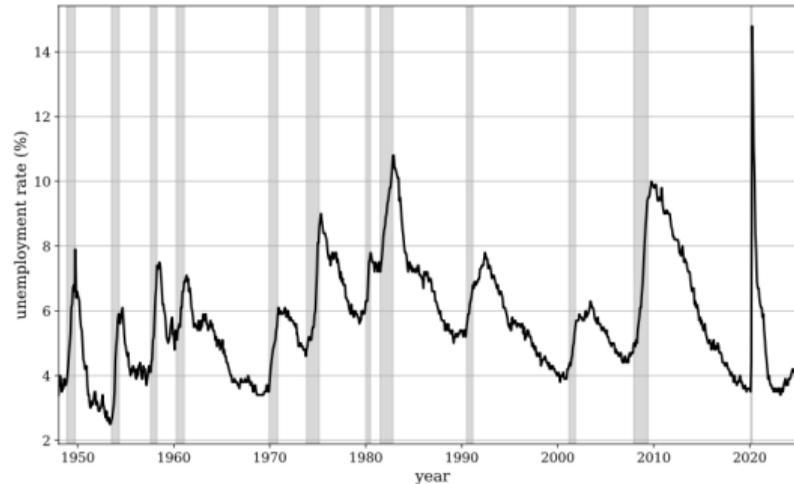


Figure 20.1: Unemployment rate in the United States.



# Unemployment & Vacancy Rates: Beveridge Curve

- **Figure 20.2:** Vacancy rate and unemployment rate (JOLTS data).
- Both vacancy  $V$  and unemployment  $U$  coexist, indicating labor market frictions.
- Negative correlation forms the Beveridge Curve.
- Provides a first glance of frictional markets: vacancies remain even with high unemployment.

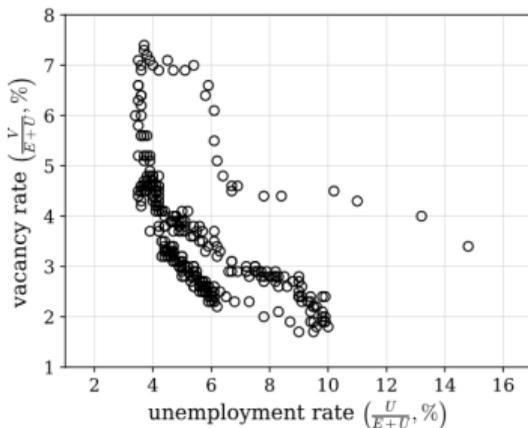


Figure 20.2: Unemployment rate and vacancy rate in the United States.



# A Two-State Model

- Workers are either **Employed (E)** or **Unemployed (U)**.
- Total labor force normalized to 1:  $e_t + u_t = 1$ .
- Each period, unemployed find jobs with probability  $\lambda$ , employed lose jobs with probability  $\sigma$ .
- Law of motion:

$$u_{t+1} = (1 - \lambda)u_t + \sigma(1 - u_t).$$



# Steady State and Duration

- In steady state,  $u_{t+1} = u_t = \bar{u}$ , solve

$$\bar{u} = \frac{\sigma}{\lambda + \sigma}.$$

- If  $\lambda$  is higher,  $\bar{u}$  falls. If  $\sigma$  is higher,  $\bar{u}$  rises.
- Average duration of unemployment  $D = 1/\lambda$  under constant  $\lambda$ .



# The Diamond-Mortensen-Pissarides (DMP) model

- A search and matching model where firms post vacancies.
- Costly vacancy posting: pay a flow cost  $\kappa$  to maintain a vacancy.
- Matches occur via a **matching function**:

$$M_{t+1} = M(u_t, v_t).$$

- Probability unemployed worker finds a job:  $\lambda_w(\theta_t)$  where  $\theta_t = \frac{v_t}{u_t}$  is market tightness.
- Probability a vacancy meets a worker:  $\lambda_f(\theta_t)$ , related by  $\lambda_w(\theta_t) = \theta_t \lambda_f(\theta_t)$ .



# Unemployment Dynamics in DMP

- Unemployment evolves as

$$u_{t+1} = (1 - \lambda_w(\theta_t)) u_t + \sigma(1 - u_t).$$

- Steady-state condition:

$$\bar{u} = \frac{\sigma}{\lambda_w(\theta) + \sigma}.$$

- Vacancy and unemployment relation yields the Beveridge curve.



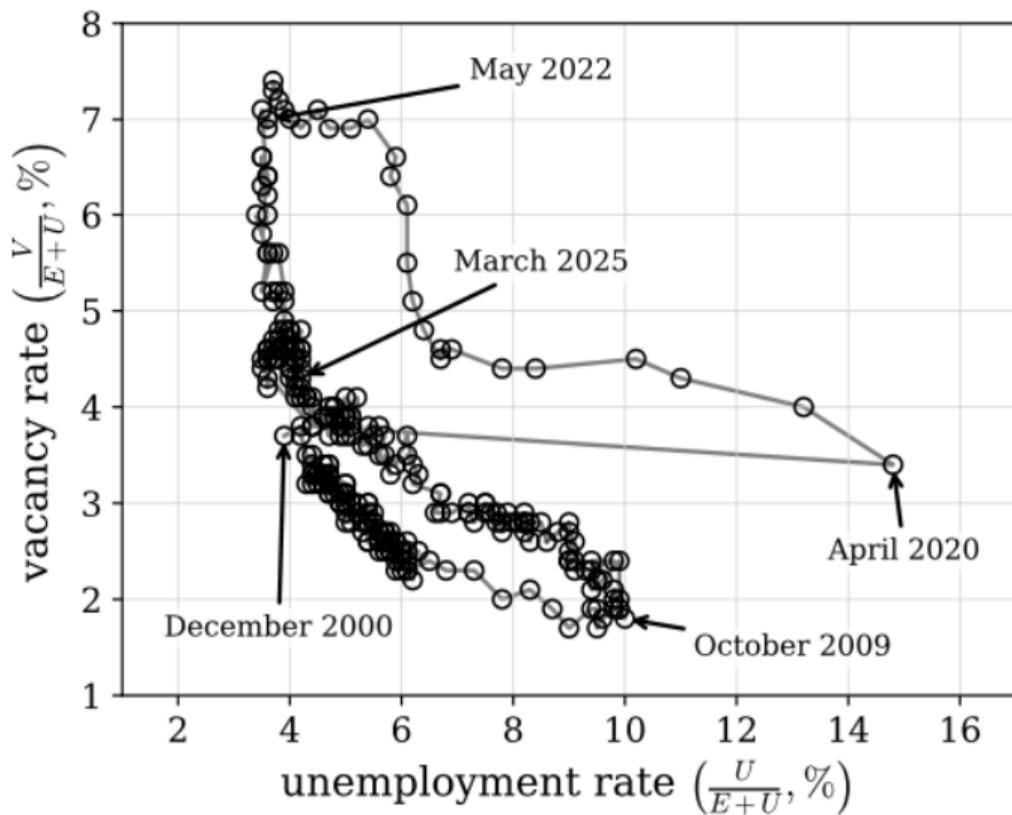


Figure 20.4: Beveridge curve in the United States.

# Vacancy Creation and Wage Setting

- Firms enter (post vacancies) until  $V(z) = 0$  in equilibrium, where  $V(z)$  is the value of a vacancy.

$$V(z) = -\kappa + \beta \mathbb{E}[\lambda_f(\theta) J(z') + (1 - \lambda_f(\theta)) V(z')].$$

- Matched firm's value  $J(z)$  depends on production  $z$ , wage  $w(z)$ , and continuation value.
- Wage  $w(z)$  is determined by **Generalized Nash Bargaining** with bargaining power  $\gamma$  for workers:

$$(1 - \gamma)(W - U) = \gamma(J - V).$$



# Job Creation Condition

- Free-entry condition implies

$$0 = V(z) \implies \kappa = \beta \mathbb{E}[\lambda_f(\theta) J(z')].$$

- Combined with the bargaining solution, we get the **Job Creation Condition**:

$$\frac{\kappa}{\lambda_f(\theta)} = \beta \mathbb{E}[z' - b + (1 - \sigma - \gamma \lambda_w(\theta')) \dots].$$

- Solves for  $\theta$  (vacancy/unemployment ratio).



# Graphical Equilibrium

- **Beveridge Curve (BC):** Negative relationship between  $v$  and  $u$ .
- **Job Creation Curve (JC):** Relationship from free-entry equilibrium.
- Intersection of BC and JC pins down  $(u, v)$  in steady state.

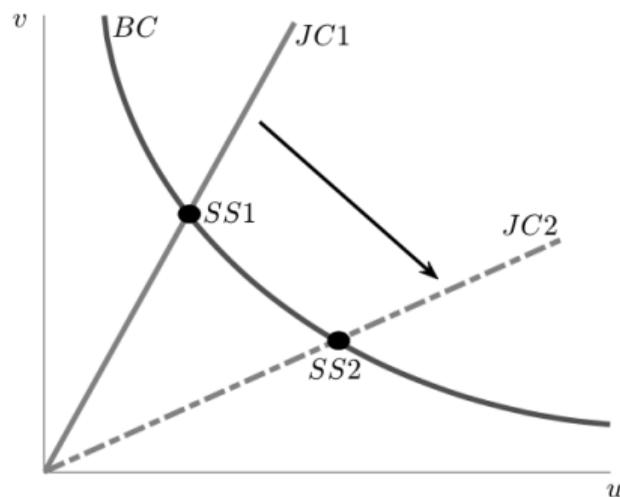


Figure 20.5: The determination of the steady state



# Efficiency and the Hosios Condition

- In general, equilibrium  $\theta$  may be socially inefficient:
  - Firms do not capture full benefit of posting vacancies (shared with workers).
  - Also create congestion externalities for other vacancies.
- **Hosios Condition:**  $\gamma = \eta(\theta)$ , where  $\eta(\theta)$  is elasticity of matching wrt. unemployment (or  $\theta$ ).
- If Hosios holds, the equilibrium is (constrained) efficient.



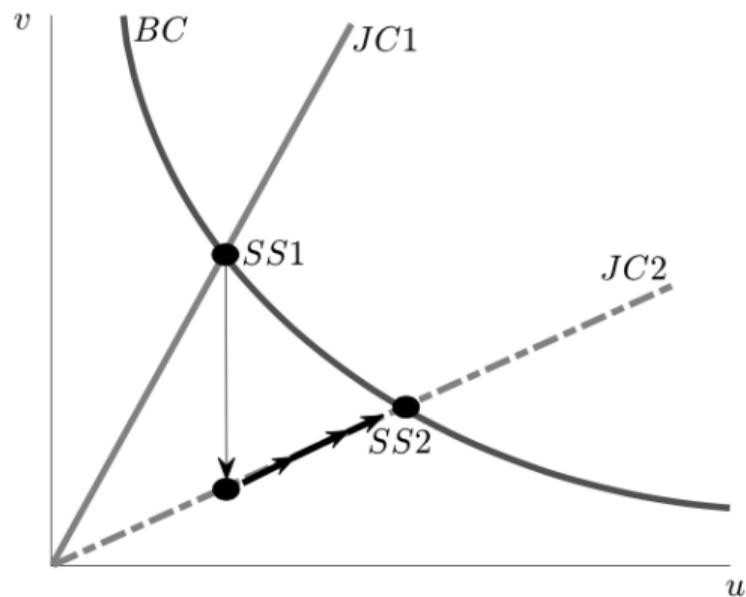


Figure 20.6: The transition dynamics of the DMP model.



# Gross Flows and Three Labor Market States

- CPS data categorizes individuals into:

$E$  (*Employed*),  $U$  (*Unemployed*),  $N$  (*Not in LF*).

- Flows among these three states are large each month.
- Both inflows and outflows of unemployment shape  $u$  over the business cycle.

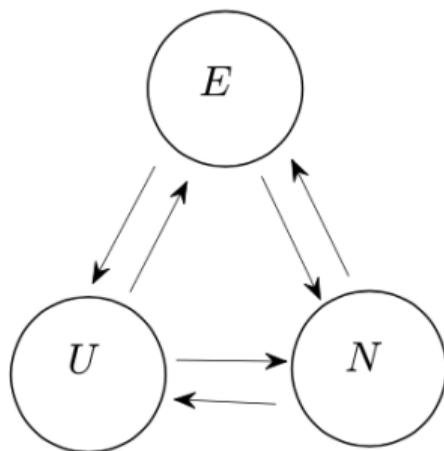


Figure 20.7: Flows among three states.



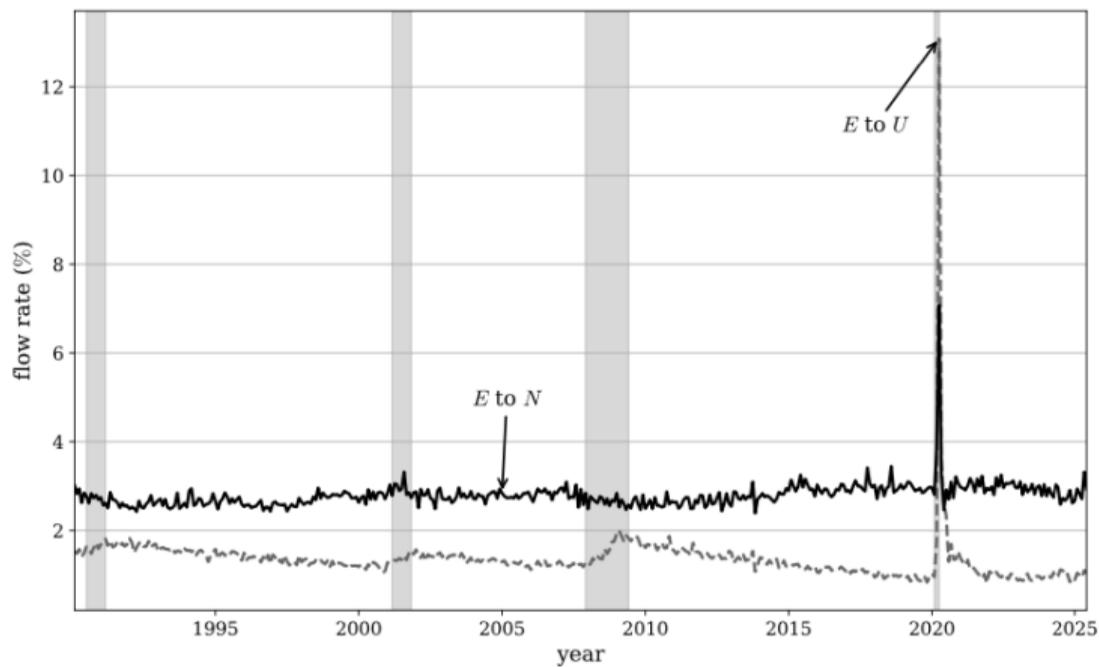


Figure 20.8: Flow rates out of  $E$ .



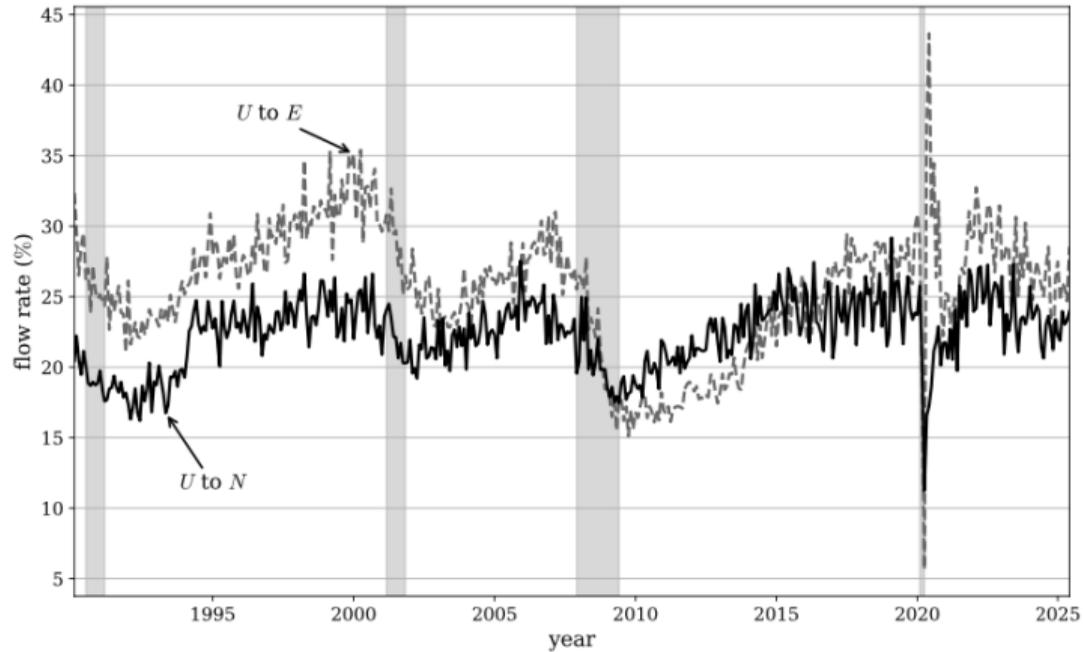


Figure 20.9: Flow rates out of  $U$ .



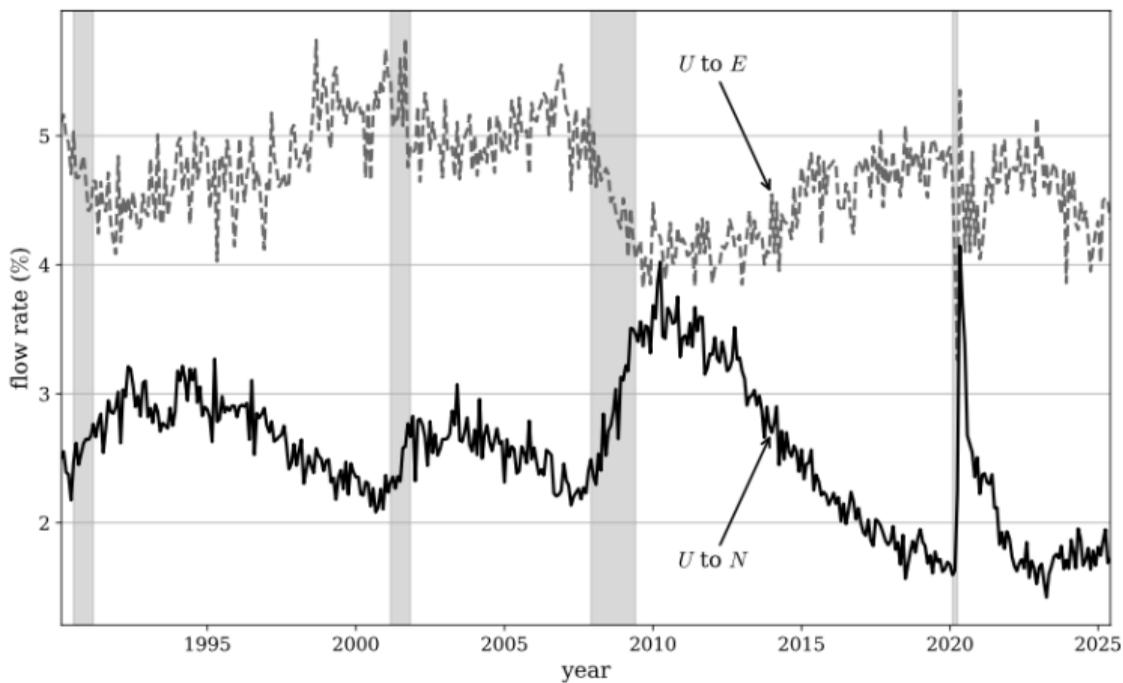


Figure 20.10: Flow rates out of  $N$ .



# Empirical Observations on Flow Rates

- Job-finding rate (U to E) is strongly procyclical.
- Separation rate (E to U) is countercyclical, rises at onset of recessions.
- Simple DMP model with constant separation  $\sigma$  misses the cyclicity in separations.



# Motivation: Small Fluctuations

- Calibrated DMP model with RBC-like shocks often underpredicts unemployment volatility.
- Shimer (2005) shows standard calibrations yield too small variance in  $u$  and  $v$ .
- Known as the **Shimer puzzle** or **unemployment volatility puzzle**.



# Potential Solution: Wage Rigidity

- In standard DMP, wages absorb productivity shocks, reducing firm profit volatility.
- If wages are rigid, profits become more procyclical, making vacancy creation more sensitive to shocks.
- (Hall, 2005) and (Shimer, 2005) highlight the importance of real wage rigidity to match volatile unemployment data.



# Separation as a Choice

- Firms can choose job destruction: pay maintenance cost  $c(\sigma)$  to lower separation probability.
- More realistic approach: separation rate is cyclical.
- Endogenizing  $\sigma$  can improve model fit, but still not enough for full volatility, unless wages are rigid.



# Model Equations

- Matched firm value:

$$J(z) = \max_{\sigma} \left\{ z - w - c(\sigma) + \beta \mathbb{E}[(1 - \sigma)J' + \sigma V'] \right\}.$$

- Optimal  $\sigma$  depends on marginal cost  $c'(\sigma)$  and continuation value.
- Job creation:

$$\frac{\kappa}{\lambda_f(\theta)} = \beta \mathbb{E}[z - b - c(\sigma) + \dots].$$



# Motivation

- Integrate labor market frictions into the RBC framework:
  - Concave utility, consumption-savings decisions.
  - Capital in production, input rented from households.
  - Endogenous unemployment via DMP frictions.
- This is often called the (Merz, 1995) or (Andolfatto, 1996) approach.



# Household (Family) Problem

- Representative family insures all members: each consumes the same amount.
- Budget constraint:

$$c_t + k_{t+1} = (1 + r_t - \delta)k_t + (1 - u_t)w_t + u_t b + d_t.$$

- Capital evolves as in RBC, with utility  $\sum \beta^t U(c_t)$ .
- Takes  $w_t, r_t, d_t$  (dividend from firms) as given from the labor market side.



# Firms and Vacancies

- Production per matched worker:  $z_t k_t^\alpha$  (shared across  $1 - u_t$  matches).
- Firms rent capital at rate  $r_t$  from households.
- Vacancy creation cost  $\kappa$ , free entry until  $V(X_t) = 0$ .
- Wages from Nash bargaining or fixed if wage rigidity is assumed.



# Job and Vacancy Value Functions

- Matched firm:

$$J(X_t) = y(X_t) - w(X_t) + \int Q(z_{t+1}, X_t) [(1 - \sigma)J(X_{t+1}) + \sigma V(X_{t+1})] dz_{t+1}.$$

- Vacancy:

$$V(X_t) = -\kappa + \int Q(z_{t+1}, X_t) [\lambda_f(\theta)J(X_{t+1}) + (1 - \lambda_f(\theta))V(X_{t+1})] dz_{t+1}.$$

- $Q(\cdot)$  is the *state price* from household's stochastic discount factor.



# Equilibrium Conditions

1. Household optimal consumption-savings choice  $\implies$  RBC Euler equation.
2. Free entry  $\implies V(X_t) = 0 \implies$  job creation condition.
3. Wage determined by either:
  - Generalized Nash Bargaining, or
  - Exogenous rigidity  $w(X_t) = \bar{w}$  (up to constraints).
4. Labor market transitions:  $u_{t+1} = (1 - \lambda_w(\theta_t))u_t + \sigma(1 - u_t)$ .



# Quantitative Results

- Similar to simpler DMP, standard calibration yields low  $u$  and  $v$  volatility.
- RBC-like features: investment is more volatile than output, consumption less so.
- Key shortfall: labor market variables remain *insufficiently* responsive.



Table 20.2: Summary statistics for quarterly U.S. data

	$u$	$v$	$v/u$	$z$
Standard Deviation	0.125	0.139	0.259	0.013
Quarterly Autocorrelation	0.870	0.904	0.896	0.765
Correlation Matrix	$u$	1	-0.919	-0.977
	$v$	—	1	0.982
	$v/u$	—	—	1
	$z$	—	—	—

Table 20.3: Model statistics

	$u$	$v$	$v/u$	$z$
Standard Deviation	0.005	0.016	0.020	0.013
Quarterly Autocorrelation	0.826	0.700	0.764	0.765
Correlation Matrix	$u$	1	-0.839	-0.904
	$v$	—	1	0.991
	$v/u$	—	—	1
	$z$	—	—	—



# Wage Rigidity in RBC + DMP

- If  $w_t$  is fixed near its steady-state value, firm's profit margin responds more to productivity shocks.
- Vacancies and thus unemployment respond more strongly, increasing  $u$  volatility.
- Achieves higher correlation with data at the cost of artificially imposed wage rigidity.



## (McCall, 1970) Search Model

- Simple model of a worker receiving one wage offer  $w$  per period, with distribution  $F(w)$ .
- Worker's reservation wage  $w^*$  solves: accept if  $w \geq w^*$ .
- Probability of getting an acceptable offer:  $1 - F(w^*)$ .



# Reservation Wage Condition

- If discount factor  $\beta$  and separation prob  $\sigma$ , then the reservation wage  $w^*$  satisfies:

$$w^* = b + \frac{\beta}{1 - \beta(1 - \sigma)} \int_{w^*}^{\bar{w}} (w - w^*) dF(w).$$

- Larger  $b$  or  $\beta \implies$  higher reservation wage  $w^*$ .
- Higher separation  $\sigma \implies$  lower  $w^*$  (less payoff to waiting).



# Frictional Wage Dispersion

- In frictionless market, all accept highest wage only  $\implies$  no dispersion.
- Here, waiting for high-wage offer is costly (risk of not getting an offer).
- Even so, with typical calibrations (high job-finding rates), the difference between the lowest accepted wage and average wage tends to be small.
- $\implies$  the **frictional wage dispersion puzzle**.



# Overview

- We introduced search frictions into labor markets to model unemployment endogenously.
- The DMP framework explains key labor market facts like the Beveridge curve.
- Standard calibrations struggle to match large unemployment fluctuations without added wage rigidity.
- Endogenous separations and wage dispersion were also examined.



# Policy Implications

- Unemployment insurance and job training are crucial but can alter  $\lambda_w$  and reservation wages.
- Design must consider efficiency vs. equity trade-offs.
- Hosios condition suggests balanced bargaining power for social efficiency in matching markets.



# Extensions and Open Questions

- On-the-job search and job-to-job transitions.
- Endogenous wage rigidities (e.g., collective bargaining, long-term contracts).
- Heterogeneity in skill levels, match-specific productivity.
- Interactions with monetary policy and nominal rigidities.



## Additional Slide: More on Job Creation Condition

- Combining free-entry ( $V(z) = 0$ ) and Nash Bargaining yields:

$$\frac{\kappa}{\lambda_f(\theta)} = \beta \mathbb{E} \left[ z' - b + (1 - \sigma - \gamma \lambda_w(\theta')) \dots \right].$$

- Captures the expected net surplus from a successful match.



# Detailed Wage Equation in DMP

$$w = b + \gamma(z - b) + \gamma\theta\kappa.$$

- The term  $\gamma(z - b)$  is the worker's share of surplus.
- Extra  $\gamma\theta\kappa$  compensates for worker's lost search opportunity.



# Efficiency and Externalities

- Negative externality: more vacancies raise congestion for other firms.
- Positive externality: more vacancies speed up matching for job-seekers.
- Hosios condition balances these to achieve efficiency:

$\gamma$  = elasticity of matching wrt. unemployment.



## Extensions: Endogenous Separation Case

- Let  $c(\sigma)$  be maintenance cost for separation rate  $\sigma$ .
- If  $c'(\sigma) < 0$ , reducing separation becomes more costly.
- Can fit cyclical separation rates better, but still requires additional wage assumptions for large  $u$  fluctuations.



# RBC + DMP Model Computation

- Solve households' consumption/saving with the standard RBC approach.
- Compute firm vacancy creation via free-entry condition.
- Clear capital market with rental rate  $r_t$  and wage from bargaining or rigid rule.
- Iterate to find a consistent equilibrium path.



# Frictional Wage Dispersion in Detail

- McCall model with wage offer distribution  $F(w)$ .
- Reservation wage  $w^*$  solves:

$$w^* = b + \frac{\beta}{1 - \beta(1 - \sigma)} \int_{w^*}^{\bar{w}} (w - w^*) dF(w).$$

- Dispersion arises because workers accept some low offers to avoid waiting.
- Empirically small difference between  $w^*$  and mean wage, unless cost of unemployment is very high.



# References I

- Andolfatto, D. (1996). Business cycles and labor-market search. *American Economic Review*, 86:112–132.
- Hall, R. E. (2005). Employment fluctuations with equilibrium wage stickiness. *American Economic Review*, 95:50–65.
- McCall, J. J. (1970). Economics of information and job search. *Quarterly Journal of Economics*, 84:113–126.
- Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics*, 36:269–300.
- Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95:25–49.



# Thank you!

Questions or comments?

[orhan.torul@bogazici.edu.tr](mailto:orhan.torul@bogazici.edu.tr)

