Chapter 21: Inequality

Graduate Macroeconomics Slides

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January 23, 2026

Outline

- Introduction and Data
- Theory: Macroeconomics and Inequality
  - Labor Share and Capital–Output Ratio
  - Wage Inequality (Skill Premium)
- Endogenous, Directed Skill-Biased Technological Change
- Capital–Skill Complementarity
- Human Capital Accumulation
- Task-Based Models
- Labor Markets in Practice
  - Wealth Inequality
- Reasons Why Inequality Matters for Aggregates
  - Long-run channels
  - The business cycle
  - From micro to macro: more heterogeneity

Motivation and scope

- Inequality has moved to the center of macro: both research and policy focus on it.
- Multiple reasons: rising concentration of earnings/wealth, general-equilibrium determination, and aggregate effects via heterogeneous marginal propensities.
- Chapter layout: data; theory of wage and wealth inequality; consequences for aggregates.

Data: cross-sectional facts

- U.S. income distribution (2022) is highly skewed: thick right tail; mean $\gg$ median.
- Similar skewness holds for earnings and wealth.
- Lorenz and Gini summarize dispersion across income/earnings/wealth.

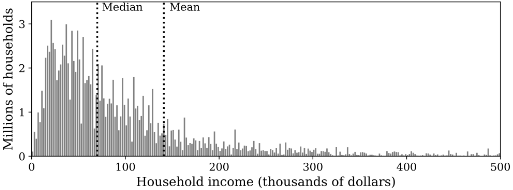


Figure 21.1: Histogram of the income distribution.

Source: Kuhn and Rios-Rull (2025) using the 2022 Survey of Consumer Finances.

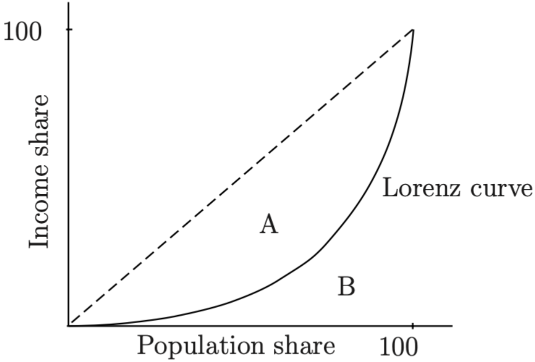


Figure 21.2: Lorenz curve

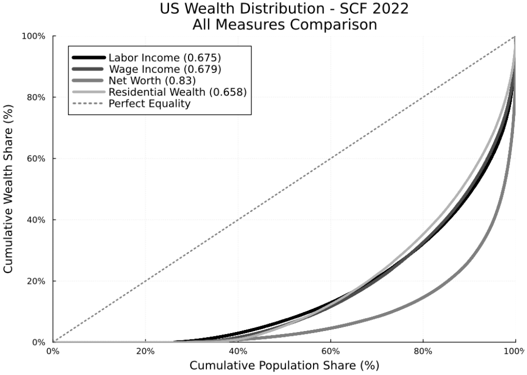


Figure 21.3: Lorenz Curves of Labor Earnings, Residential and Total Wealth

Distributions of consumption and hours

- Consumption dispersion is much lower than dispersion in earnings, income, and wealth (permanent income + private insurance).
- Hours dispersion is modest (especially per person); systematic patterns are limited.
- Residualized hours are evenly distributed across wage ventiles; hours have fallen for the wage-poorest in recent decades.

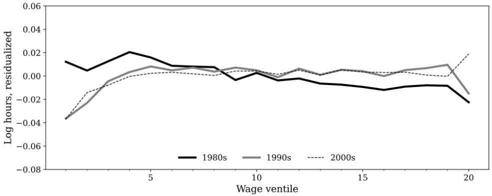


Figure 21.4: (Log) hours worked by wage ventiles.

Note: Ventiles are 5% bins. **Source:** Boppart et al. (2024).

Wealth-to-income and capital-to-GDP ratios; trends

- Wealth-to-income and K/Y have risen; wealth gains concentrated at the top.
- Skill premium rose strongly after 1980 despite higher college supply.

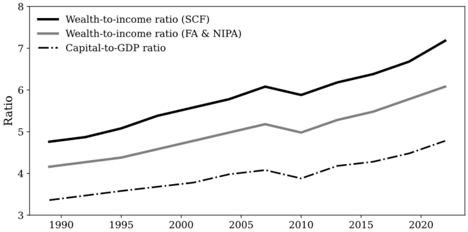
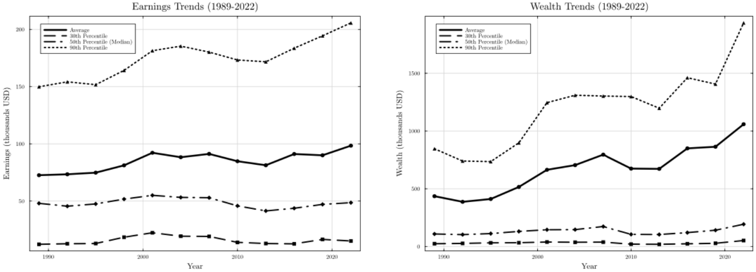


Figure 21.7: Wealth-to-income and capital stock-to-GDP ratios by SCF wave.

Note: Wealth-to-income ratio based on the SCF, wealth-to-income ratio using income from National Income and Product Accounts (NIPA), and wealth from Financial Accounts (FA), and capital stock-to-GDP ratio from Penn World Table 10.01. **Source**: Kuhn and Rios-Rull (2025).



(a) Earnings trends

(b) Wealth Trends

Figure 21.8: Inequality statistics for the evolution of earnings and wealth 1989–2022

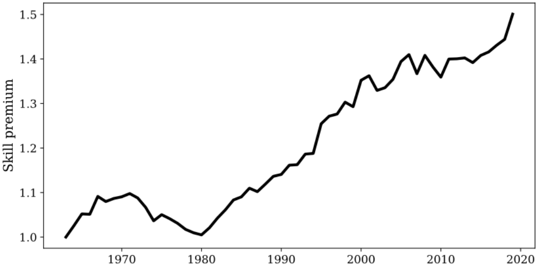


Figure 21.9: Evolution of the skill premium over time.

Source: Ohanian et al. (2023).

Figure: (Ohanian et al., 2023)

Portfolios and returns across the wealth distribution

- Portfolio composition varies systematically with wealth: cash at bottom, housing in middle, risky assets and private equity at top.
- Mean portfolio returns rise with wealth percentile; risk also changes.

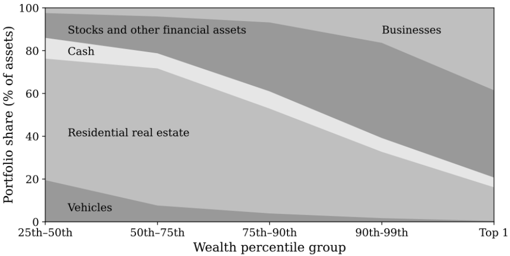


Figure 21.5: Portfolio shares from US Survey of Consumer Finance, 2022.

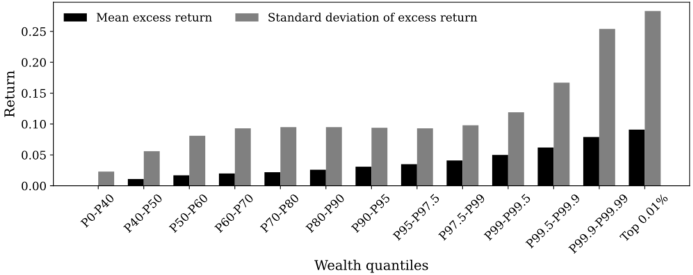


Figure 21.6: Mean and standard deviation for excess returns on portfolios.

Shares table across distributions (2022)

- The top 1% holds a very large share of wealth; consumption shares are much flatter.
- Hours shares do not concentrate like earnings/wealth shares.

Table 21.1: 2022 Per household shares of selected groups sorted by each variable

	Bottom			Quintiles					Top			
	0-1	1-5	5-10	0-20	20-40	40-60	60-80	80-100	90-95	95-99	99-100	Gini
Earnings	−0.00	0.00	0.00	−0.16	0.50	10	0.96	3.39	2.5	4.88	19.4	0.68
Income	0.00	0.08	0.12	0.14	0.30	0.50	0.82	3.24	2.5	4.45	22.4	0.61
Wealth	−0.2	−0.02	0	−0.01	0.05	0.19	0.50	4.70	2.48	6.48	35.1	0.83
Consumption				0.44	0.66	0.84	1.12	1.93				
Hours worked												
per household	0.07	1.05	2.25	9.31	12.64	16.84	23.54	37.66	8.18	8.04	7.65	
per person	0.09	1.28	2.65	11.65	19.89	20.57	20.97	26.91	6.67	6.27	2.07	

Note: Shares of the earnings, income, and wealth distribution in 2022. For earnings, income and wealth the source is Kuhn and Rios-Rull (2025). For consumption data from Table 1101, https://www.bls.gov/cex/tables/calendar-year/aggregate-group-share/cu-income-quintiles-before-taxes-2022.pdf.

Figure: (Kuhn and Ríos-Rull, 2025)

Labor share with factor-augmenting technology

Consider a constant-returns production function $F(A_k k, A_h h)$ with competitive factor markets. The capital share equals

$$\frac{r_t k_t}{y_t} = \frac{A_{k,t} F_k(A_{k,t} k_t, A_{h,t} h_t) k_t}{F(A_{k,t} k_t, A_{h,t} h_t)} = F_k\left(1, \frac{A_{h,t} h_t}{A_{k,t} k_t}\right) \Big/ F\left(1, \frac{A_{h,t} h_t}{A_{k,t} k_t}\right),$$

by homogeneity. Hence the share depends on the effective input ratio $A_{h,t} h_t / (A_{k,t} k_t)$ and may vary off balanced growth.

If F is CES with elasticity ρ, the share admits a closed form in that ratio; declining labor shares can arise from shifts in (A_k, A_h) or movements in k/h outside steady state.

CES accounting for the skill premium

Let production embed a CES sub-aggregate of skilled S and unskilled U efficiency units:

$$Y = F(K,\ G(A_S S, A_U U)), \qquad G(x,y) = \left[\alpha x^{\frac{\sigma-1}{\sigma}} + (1-\alpha) y^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

with elasticity of substitution $\sigma > 0$. Competitive wages satisfy

$$\frac{w_S}{w_U} = \frac{\partial G/\partial(A_S S)}{\partial G/\partial(A_U U)} = \left(\frac{A_S}{A_U}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{S}{U}\right)^{-\frac{1}{\sigma}}.$$

Thus $\log(w_S/w_U) = \frac{\sigma-1}{\sigma}\log(A_S/A_U) - \frac{1}{\sigma}\log(S/U)$, clarifying the roles of skill-biased efficiency growth and relative supply. *Empirically, the skill premium rises since 1980 despite increased S/U, consistent with A_S/A_U growth.*

Skill-Biased Technological Change Setup

Consider an aggregator for effective labor,

$$\mathcal{G}(A_s s, A_u u) = \Big[(A_s s)^\sigma + (A_u u)^\sigma\Big]^{1/\sigma}, \qquad \sigma > 0,$$

and a technology-choice (R&D) constraint,

$$\Big[\lambda A_s^\phi + (1-\lambda) A_u^\phi\Big]^{1/\phi} = 1, \qquad \phi > 1.$$

The social problem chooses $\{A_s, A_u\}$ to maximize $\mathcal{G}$ subject to the R&D constraint; $\lambda \in (0,1)$ captures the research weight directed to skill-specific improvements. This is a tractable version of directed SBTC as in Acemoglu (1998).

First-order conditions and directedness

Let $\mathcal{L} = \left[(A_s s)^\sigma + (A_u u)^\sigma\right]^{1/\sigma} - \mu\Big(\left[\lambda A_s^\phi + (1-\lambda)A_u^\phi\right]^{1/\phi} - 1\Big)$. Differentiating and using the envelope of the CES yields

$$\frac{\partial \mathcal{L}}{\partial A_s} \propto (A_s s)^{\sigma-1} s - \mu\, \lambda^{1/\phi} A_s^{\phi-1}, \qquad \frac{\partial \mathcal{L}}{\partial A_u} \propto (A_u u)^{\sigma-1} u - \mu\, (1-\lambda)^{1/\phi} A_u^{\phi-1}.$$

Dividing the two FOCs gives the relative efficiency ratio

$$\boxed{\frac{A_s}{A_u} = \frac{\lambda}{1-\lambda}\left(\frac{s}{u}\right)^{\frac{\sigma}{\phi-\sigma}}}$$

for $\phi \neq \sigma$. A larger relative supply s/u attracts R&D toward the skill used more intensively, endogenously raising A_s/A_u when $\phi > \sigma$.

Implication for the skill premium

With competitive factor prices from $\mathcal{G}$,

$$\frac{w_s}{w_u} = \frac{\partial\mathcal{G}/\partial(A_s s)}{\partial\mathcal{G}/\partial(A_u u)} = \left(\frac{A_s}{A_u}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{s}{u}\right)^{-\frac{1}{\sigma}}.$$

Substituting $\frac{A_s}{A_u}$ from the FOC yields

$$\log\frac{w_s}{w_u} = \left[\frac{\sigma-\phi(1-\sigma)}{\phi-\sigma}\right]\log\frac{s}{u} + \frac{\sigma-1}{\sigma}\log\frac{\lambda}{1-\lambda}.$$

Hence the skill premium is *increasing* in s/u when

$$\boxed{\frac{\sigma-\phi(1-\sigma)}{\phi-\sigma} > 0},$$

which obtains for sufficiently high elasticity between s and u; empirically, values above ≈ 2 can overturn the neoclassical dilution effect, making a college boom raise the premium via directed change (Acemoglu, 1998).

Directed SBTC: intuition and empirical link

- R&D is reallocated toward the larger market: a rise in s/u raises the payoff from A_s, increasing A_s/A_u.
- The premium combines two forces: supply dilution $-\frac{1}{\sigma}\log(s/u)$ and efficiency tilt $+\frac{\sigma-1}{\sigma}\log(A_s/A_u)$.
- When substitution between skilled and unskilled is high and the R&D trade-off is not too steep, the efficiency tilt dominates.
- This mechanism rationalizes a rising skill premium since the 1980s despite a growing college share.

A simple example

Consider $F(k, u, s) = u + s^{\nu} k^{1-\nu}$ with $\nu \in (0, 1)$. Marginal products are

$$\frac{\partial F}{\partial s} = \nu s^{\nu-1} k^{1-\nu}, \qquad \frac{\partial F}{\partial u} = 1.$$

Capital deepening ($k \uparrow$) raises the marginal product and wage of skilled labor while leaving the unskilled marginal product unchanged. The skill premium w_s / w_u therefore increases with k. This illustrates capital–skill complementarity highlighted by Krusell et al. (2000).

General CES nesting and qualitative predictions

- Nest skilled labor with capital: $F\big(u,\ G(k,s)\big)$ with a higher elasticity inside G than between u and G. Rapid growth in k then lifts the relative wage of skilled labor even without explicit trends in (A_s, A_u) (Griliches, 1969).
- The alternative nest $F\big(s,\ G(k,u)\big)$ allows $k\uparrow$ to depress w_u; this is consistent with stagnant low-skill wages in several decades.
- Capital may be especially complementary with human capital in times of rapid technological change (Greenwood and Yorukoglu, 1997).

Concepts and sorting

- Human capital can be *general* or *specific*; it is multi-dimensional and not always transferable across tasks or occupations (Roy selection).
- Sorting of heterogeneous workers into jobs maximizes income given skill vectors; complementarities between skills and capital equipment generate systematic matching patterns.
- Markets and institutions affect the mapping from skills to wages; liquidity constraints and incomplete credit markets intertwine education and consumption choices, creating potential low-skill traps (Griffy, 2021).

Ben-Porath life-cycle problem

A worker chooses consumption c_t and human-capital investment i_t to maximize

$$\max_{\{c_t, i_t\}} \int_0^T e^{-\rho t} u(c_t)\, \mathrm{d}t \quad \text{s.t.} \quad \dot{h}_t = \phi(i_t) - \delta h_t, \; c_t + i_t \leq w_t h_t + r a_t, \; \dot{a}_t = r a_t + w_t h_t - c_t - i_t.$$

with h_0 given. First-order conditions yield $u'(c_t) = \lambda_t$, $\phi'(i_t) = q_t$, and the co-state system $\dot{q}_t = (\rho + \delta - \dot{w}_t / w_t)\, q_t - \lambda_t$; the solution features early-life investment, mid-life interior trade-off, and late-life specialization in work (Ben-Porath, 1967). With imperfect credit, the Euler equations couple to liquidity, affecting schooling timing and level.

From tasks to aggregate production

- A production process is a set of complementary tasks. Inputs (labor types, capital) have task-specific productivities.
- Assignment solves a cost-minimization over tasks; under suitable assumptions, aggregate output $Y = F(K, \{L_j\})$ is derived endogenously from the task technology.
- The framework gives concrete language for *automation*, *hollowing out*, *robots*, and *offshoring*: tasks previously done by mid-skill workers become capital- or software-performed.
- Empirics map O*NET task content (abstract, routine, manual) to factor intensities and trace shifts over time.

Beyond $w = MPL$: mechanisms

- **Search frictions.** Workers with identical observables can receive different wages due to search and matching; large residual dispersion remains even after controlling for observables.
- **Compensating differentials (amenities).** Jobs differ in safety, flexibility, and remote options; selection effects matter for welfare comparisons as workers sort on amenity preferences.
- **Monopsony power and unions.** Concentration and mobility frictions give firms wage-setting power; union coverage (and collective bargaining spillovers) alters wage floors and inequality patterns.

Deterministic benchmark (dynastic model)

Consider a frictionless steady state without aggregate growth. Consumer i solves

$$\max_{\{c_{i,t}, a_{i,t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \quad \text{s.t.} \quad c_{i,t} + a_{i,t+1} = (1 + r - \delta) a_{i,t} + \varepsilon_i w,$$

with $\sum_i \varepsilon_i = 1$, labor supplied inelastically, and $K_t = \sum_i a_{i,t}$ in equilibrium. The Euler condition pins down the steady-state interest rate; any long-run wealth distribution can arise and depends on initial wealth. This motivates introducing frictions and risk to discipline the distribution.

Incomplete markets ((Aiyagari, 1994)

In a canonical HA model, an individual with assets a and idiosyncratic income state z solves

$$V(a,z) = \max_{c,a'} \{u(c) + \beta \mathbb{E}[V(a',z')|z]\} \quad \text{s.t.} \quad c + a' = wz + (1+r)a, \;\; a' \geq \underline{a},$$

with z Markov and borrowing limit $\underline{a}$. Optimal policy $a' = g(a,z)$ and the income process induce a stationary wealth distribution μ. General equilibrium chooses (r,w) such that factor markets clear given μ. The model accounts for skewed wealth via self-insurance, borrowing limits, and precautionary saving, but benchmark versions may under-produce the extreme left and right tails without additional mechanisms.

Portfolios, returns, and wealth concentration

Heterogeneous portfolios generate heterogeneous realized returns: cash-heavy at the bottom, housing in the middle, risky financial wealth at the top, with higher mean returns for the wealthy. Over time, this return heterogeneity amplifies wealth dispersion, consistent with registry evidence and cross-country data on component returns.

Why inequality matters for aggregates

- Inequality can reflect incentives to work, save, and innovate; equalizing consumption via proportional taxes plus lump-sum transfers compresses dispersion but lowers K and hours in steady state.
- With missing insurance markets, redistribution can raise welfare ex-ante, but deeper frictions (private information, lack of commitment) constrain feasible policies.
- Innovation and risk-sharing interact: poor insurance can shift innovation to organizations that pool risks.

MPC heterogeneity, wealthy hand-to-mouth, and demand

- In HA models, high and dispersed MPCs amplify the effects of transfers and income shocks on consumption.
- Wealthy hand-to-mouth arises with illiquid portfolios; a simple device is a random constraint $a' \geq a$ preventing decumulation in some periods, pushing MPCs toward 1 when income is low.
- Barebones Aiyagari lacks a strong demand side; matching output comovement often requires reallocation frictions or New Keynesian features.

Demographics, households, search, and geography

- Age structure, fertility, and pensions affect aggregate saving and labor supply.
- Marriage/cohabitation alters participation elasticities ("second earners") and is countercyclical.
- Education, occupation, experience, health, and location interact with search/matching; distributions over matches can matter for dynamics.

CES labor share formula

With $F(x,y) = \left[\alpha x^{\frac{\rho-1}{\rho}} + (1-\alpha)y^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$ and $x = A_k k$, $y = A_h h$,

$$\text{Cap. share} = \frac{rk}{y} = \frac{\partial F}{\partial x}\frac{x}{F} = \frac{\alpha\left(\frac{x}{F}\right)^{\frac{1}{\rho}}}{\alpha\left(\frac{x}{F}\right)^{\frac{1}{\rho}} + (1-\alpha)\left(\frac{y}{F}\right)^{\frac{1}{\rho}}} = \frac{1}{1 + \frac{1-\alpha}{\alpha}\left(\frac{y}{x}\right)^{\frac{1}{\rho}}}.$$

Hence movements in the effective ratio $y/x = (A_h h)/(A_k k)$ drive factor share dynamics off the balanced path.

Stationary wealth distribution and market clearing

Let Γ denote the Markov operator induced by policy g and income process for the distribution of (a, z). A stationary distribution μ solves $\mu = \Gamma(\mu)$. Capital market clearing requires $K(\mu) = \int a\, \mu(\mathrm{d}a, \mathrm{d}z)$ equals the firm's K that delivers the equilibrium (r, w). Iterating on $(r, w) \leftrightarrow \mu$ gives a fixed point. Benchmark models match skewness but under-produce mass at very low wealth without a consumption floor or additional frictions.

Welfare comparisons under transitions

Dynamic policy evaluation must account for transition paths and heterogeneous impacts. Correct comparisons compute present-value utility at time 0 along the full transition for each agent, possibly reporting a distribution of gains and losses rather than a single number.

Modeling frictions and the Lucas critique

Canonical HA models often treat missing markets as exogenous; policy analysis must consider that contracts would adjust. Lucas-proof approaches embed private information and commitment frictions to derive optimal contracts, but they are technically demanding and sometimes empirically remote.

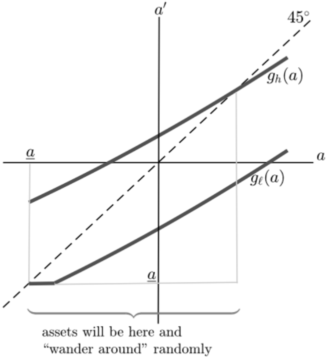


Figure 21.10: MPCs high for low asset values (1 when constraint binds).

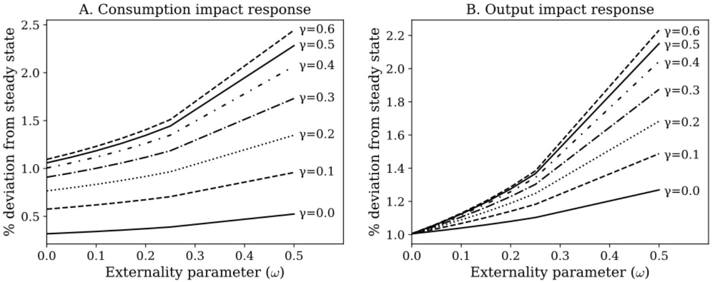


Figure 21.11: Impact responses to a 1 percent exogenous TFP shock.

Summary

- Distributions of income/earnings/wealth are highly skewed; consumption and hours are much flatter.
- Skill premium rose strongly post-1980; wealth and K/Y trended up, with top-heavy gains.
- Labor share dynamics can be understood with factor-augmenting CES and off–balanced-path movements.
- Wealth inequality requires incomplete markets, borrowing limits, return heterogeneity, and possibly additional mechanisms for tails.
- Inequality affects aggregates via MPC heterogeneity, illiquid portfolios, and policy transmission; a demand side is needed for output comovement.

References I

Acemoglu, D. (1998). Why do new technologies complement skills? directed technical change and wage inequality. *The quarterly journal of economics*, 113(4):1055–1089.

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684.

Ben-Porath, Y. (1967). The production of human capital and the life cycle of earnings. *Journal of Political Economy*, 75(4):352–365.

Greenwood, J. and Yorukoglu, M. (1997). 1974. *Carnegie-Rochester Conference Series on Public Policy*, 46(2):49–95.

Griffy, B. S. (2021). Search and the sources of life-cycle inequality. *International Economic Review*, 62(4):1321–1362.

Griliches, Z. (1969). Capital-skill complementarity. *Review of Economics and Statistics*, 51(4):465–468.

Krusell, P., Ohanian, L. E., Ríos-Rull, J.-V., and Violante, G. L. (2000). Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica*, 68(5):1029–1054.

Kuhn, M. and Ríos-Rull, J.-V. (2025). Income and wealth inequality in the united states: An update including the 2022 wave. *Federal Reserve Bank of Minneapolis Quarterly Review*. Forthcoming.

Ohanian, L. E., Orak, M., and Shen, S. (2023). Revisiting capital-skill complementarity, inequality, and labor share. *Review of Economic Dynamics*, 51:479–505.

Thank you!

Questions or comments?

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