# Chapter 22: Heterogenous Firms
## Graduate Macroeconomics Slides

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January 23, 2026

# Outline

- Introduction
- A Simple Model of Heterogeneous Firms
- Firm Heterogeneity in the Data
- Reallocation and Misallocation
- Firm Heterogeneity in General Equilibrium
- Alternative Market Arrangements
- Business Cycles and Heterogeneous Firms
- Endogenous Productivity
- Conclusion

# Motivation

**Classical assumption:** Aggregate production function $Y = F(K, L)$ and homogeneous firms.

**Reality:** Firms vary in size, age, productivity, growth, etc.

- Large vs. small firms
- Young vs. old
- Productive vs. unproductive

**Question:** How does firm heterogeneity affect macroeconomic outcomes?

# Motivation (continued)

**Policy relevance:**

- Entry barriers: Should we encourage or discourage new firms?
- Growth policies: How to support fast-growing, more productive firms?
- Dominance of mega-firms: Effects on competition, labor share, and business cycles.
- Decline in business dynamism: Slowing entry and reallocation across firms.

Aggregate production function alone cannot capture these nuances.

# Key Topics of the chapter

1. A Simple Model of Heterogeneous Firms
2. Empirical Evidence: U.S. Firm-Level Data
3. Reallocation and Misallocation
4. A General Equilibrium Model with Firm Heterogeneity
5. Alternative Market Arrangements (Monopolistic competition, Oligopoly)
6. Business Cycle Implications
7. Endogenous Productivity (Klette-Kortum model)

# Setup: Basic Structure

- A continuum of firms $i \in [0, 1]$.
- Production function of firm $i$:

$$y_i = a_i F(x_i)^{\gamma}, \quad \gamma \in (0, 1).$$

- $a_i$ is firm-specific productivity (heterogeneous).
- $F(x_i)$ is constant returns to scale, but $F(x_i)^{\gamma}$ has decreasing returns overall.

# Optimal Input Allocation

**Firm's cost-minimization (two steps):**

1. Minimize cost subject to $F(x_i) = 1$.

$$\min_{x_i}\{p \cdot x_i : F(x_i) = 1\} \quad \Longrightarrow \quad x_i^* \quad \text{and} \quad c \equiv p \cdot x_i^*$$

2. Scale of production:

$$\max_{m_i}\left\{a_i\, m_i^\gamma \;-\; c\, m_i\right\}.$$

**Key result:** Equilibrium aggregate production can be written as

$$Y = A\, F(X)^\gamma,$$

where $A$ depends on the distribution of $\{a_i\}$.

# Role of Dispersion in $a_i$

- If $\ln(a_i)$ is lognormally distributed with variance $\sigma^2$, then

$$A = \exp\left(\nu + \frac{\gamma}{1-\gamma}\frac{\sigma^2}{2}\right).$$

- Higher $\sigma^2$ (greater dispersion) $\implies$ higher $A$.
- **Intuition:** Productive firms (in the right tail of $a_i$) get allocated more inputs, boosting aggregate productivity beyond the mean.

# Firm Size Distribution (U.S.)

- Over 5 million firms in the U.S.
- Most firms are very small (1–4 employees).
- Large firms (over 1,000 employees) still number in thousands.
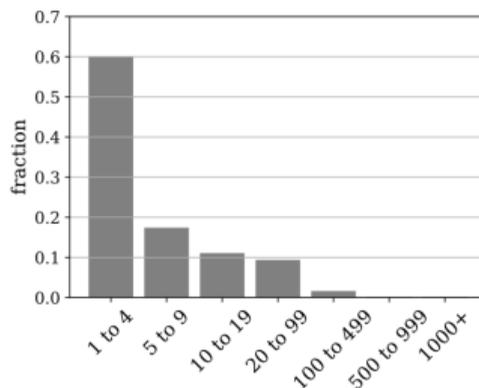


Figure 22.1: Distribution of firm size in 2022.

**Source**: Business Dynamics Statistics.

Figure: Hypothetical representation of the 2019 BDS firm size distribution. (Replace with actual figure if desired.)

# Employment Share by Firm Size

- Despite many small firms, large firms account for a substantial share of total employment.
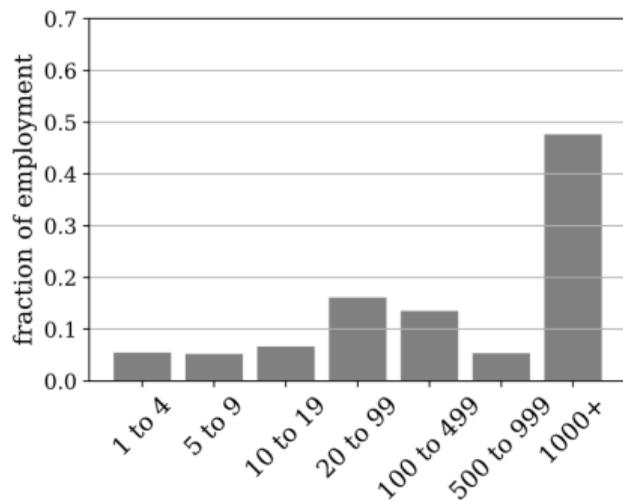- Firms with 1,000+ employees cover roughly half of U.S. employment.



Figure 22.2: Employment share of each size category in 2022.

**Source**: Business Dynamics Statistics.

# Business Dynamism: Job Creation and Destruction

- Large annual turnover rates (job creation and job destruction).
- **Cyclicality:** During recessions, job destruction rises, job creation falls.
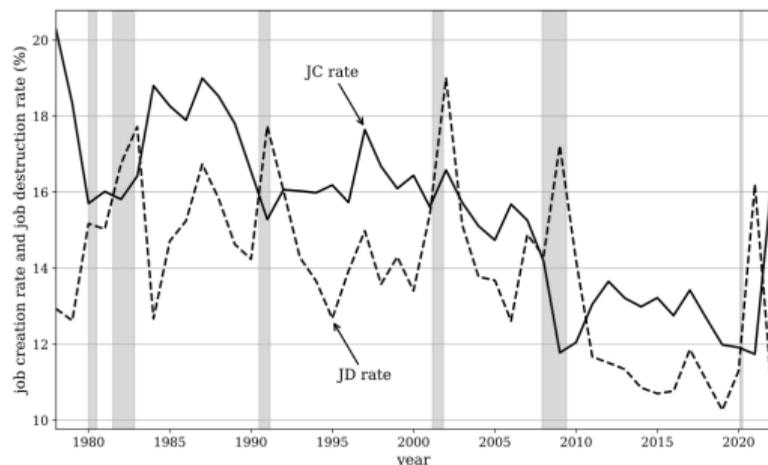- **Declining trend** in both since the 1980s: the so-called "decline in business dynamism".



Figure 22.5: Annual job creation and destruction rates (establishment level).

**Source**: Business Dynamics Statistics.

# Entry and Exit Rates

- Similar cyclicality and declining trend in establishment entry/exit rates.
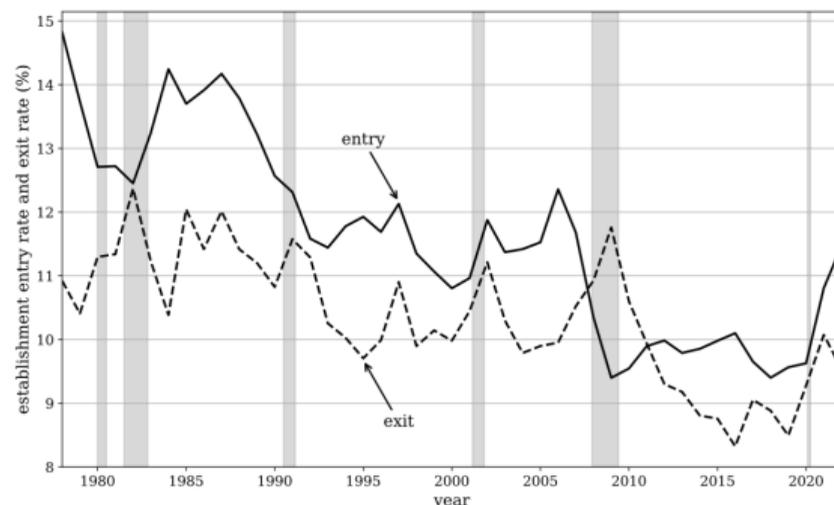- $\implies$ Less reallocation activity across businesses.



Figure 22.6: Annual establishment entry and exit rates.

**Source**: Business Dynamics Statistics.

Figure: Establishment entry and exit rates. (BDS)

# Rise of Mega-Firms & Market Power

- Large firms (10,000+ employees) are growing in share (now ~30% of total employment).
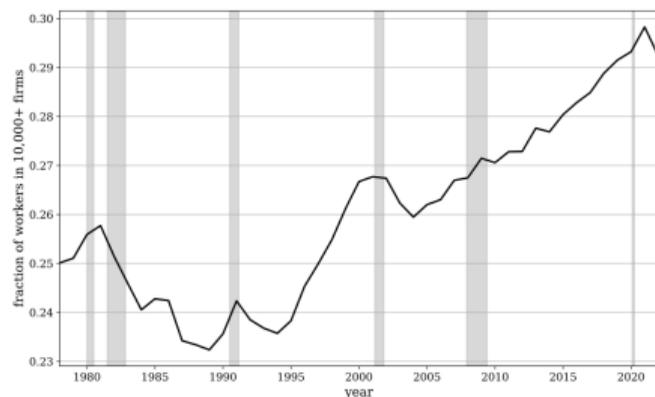- Empirical evidence of increasing average markups since the 1980s (De Loecker et al., 2020).



Figure 22.7: Fraction of employees working in firms with 10,000+ employee.

**Source**: Business Dynamics Statistics.
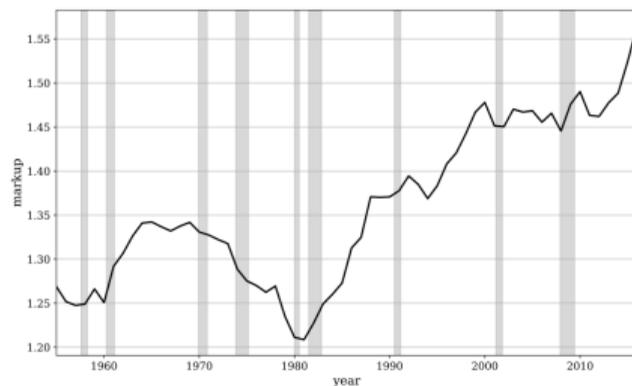


Figure 22.8: Markup of the U.S. public firms.

**Source**: De Loecker et al. (2020).

Figure: Fraction of employees at 10,000+ firms (BDS data) and average markups.

# Reallocation & Productivity

(Foster et al., 2001): Decomposition

$$\Delta \bar{A}_t = \sum_{i \in C} s_{i,t-1} \Delta a_{it} \; + \; \sum_{i \in C} (a_{i,t-1} - \bar{A}_{t-1}) \, \Delta s_{it} \; + \; \dots$$

- Reallocation effects (shifts in market shares) account for **55%** of manufacturing productivity growth in U.S. (1977–1987).
- Highlights the importance of how resources move across firms.

# Misallocation

- If distortions (e.g., taxes/subsidies) vary by firm, resource allocation is suboptimal.
- Example: (Restuccia and Rogerson, 2008), (Hsieh and Klenow, 2009).
- Leads to lower aggregate $A$ ("misallocation").

### Simple illustration

$$\max_{m_i} \sum_i (1 - \tau_i)\, a_i m_i^{\gamma} - c\, m_i.$$

Distortions $\tau_i$ shrink aggregate output if they systematically reduce $m_i$ for the most productive $a_i$.

# (Hopenhayn and Rogerson, 1993) Framework

**Key elements:**

- Continuum of firms, each with idiosyncratic productivity $a_t$.
- Firing tax $\tau \max(0, l_{t-1} - l_t)$.
- Fixed operation cost $c_f$, free entry with cost $c_e$.
- Block-recursive structure: wage $w$ determined from free-entry + labor supply.

# Stationary Equilibrium

**Firm problem:**

$$W(a, l_{t-1}) = \max_{l_t} \left[ a\, l_t^{\gamma} - w\, l_t - c_f - \tau \max(0, l_{t-1} - l_t) \right] + \beta\, \mathbb{E}[W(a', l_t)].$$

- Free entry: $W^e = c_e + \kappa$ (policy cost).
- In equilibrium, $w$ solves labor market clearing with $L_d(w) = L_s(w)$.

# Quantitative Effects of Firing Taxes

- Calibrate $\tau = 0$ baseline to match U.S. data (exit rate, size distribution).
- Increase $\tau$ from 0 to 0.1, 0.2 (equivalent to months of wages).
- **Outcome:** Lower employment, lower labor productivity, reduced job creation and destruction.

|  | $\tau = 0$ | $\tau = 0.1$ | $\tau = 0.2$ |
|---|---|---|---|
| $w$ | 1.00 | 0.977 | 0.957 |
| Total output | 100 | 97.7 | 95.7 |
| Total employment | 100 | 98.3 | 97.4 |
| Labor productivity | 100 | 99.4 | 98.3 |
| JC rate(%) | 28 | 25 | 21 |

# Effect of Entry Barriers

- Introduce extra entry cost $\kappa$.
- Raises firm value $\implies$ lowers wage $\implies$ more low-productivity firms survive $\implies$ lowers aggregate productivity.

|                    | $\kappa = 0$ | $\kappa = 0.5$ | $\kappa = 5.0$ |
|--------------------|--------------|----------------|----------------|
| $w$                | 1.00         | 0.986          | 0.879          |
| Total output       | 100          | 98.6           | 87.9           |
| Total emp.         | 100          | 99.5           | 96.2           |
| Labor productivity | 100          | 99.1           | 91.4           |
| JC rate(%)         | 28           | 28             | 28             |

# Monopolistic Competition

- Differentiated intermediate goods, final good has CES aggregator.
- With elasticity of substitution $\sigma > 1$, each producer faces demand:

$$p_i = \left(\frac{y_i}{Y}\right)^{-1/\sigma}.$$

- Constant markup: $\frac{\sigma}{\sigma-1}$ in basic CES case.
- Heterogeneity $\implies$ different $a_i$ but same markup with standard CES assumption.

# Oligopoly with Endogenous Markups

**Motivation:** (De Loecker et al., 2020) find rising markups $\implies$ Markups may vary endogenously with competition.

**Atkeson & Burstein (2008) approach:**

- Within-sector competition among $J$ brands, across-sector elasticity $\sigma$.
- If $J$ large, markup is close to $\sigma/(\sigma - 1)$; if $J = 1$ (monopoly), markup can be larger.
- Firm-level sales share $s_{ij}$ determines elasticity $\varepsilon(s_{ij}) \implies$ markup changes with share.

# Aggregate Shocks vs. Idiosyncratic Shocks

- Standard result: Idiosyncratic shocks tend to wash out in aggregate (Law of Large Numbers).
- $\text{Var}(Y) \approx \frac{\text{Var}(y_i)}{N}$ if $\{y_i\}$ are i.i.d. with finite variance.
- Not enough to generate big business cycles in standard large economy with many small firms.

# "Granular" Effects: Large Firms

**(Gabaix, 2011):** If firm size distribution is *fat-tailed* (Pareto with shape near 1), large idiosyncratic shocks can affect aggregate outcomes significantly.

**Key insights:**

- Distribution with Pareto tail $\implies$ top firms weigh heavily.
- Many real-world large firms (e.g., Amazon, Walmart) have huge sales $\implies$ big Domar weights $\implies$ big macro impact.

# Production Networks

**(Hulten, 1978) Theorem**:

$$\frac{dY}{Y} = \sum_i D_i \frac{da_i}{a_i}, \quad D_i = \frac{p_i\, y_i}{\sum_j p_j\, c_j}.$$

- Domar weight $D_i$ depends on *sales*, not just value-added.
- Downstream firms with large sales magnify shocks through the network.

# (Klette and Kortum, 2004) Model

- Continuum of products, each produced by some firm.
- Innovation $\implies$ new generation of product with higher quality $\lambda > 1$.
- Incumbents and entrants compete in R&D to take over product lines.
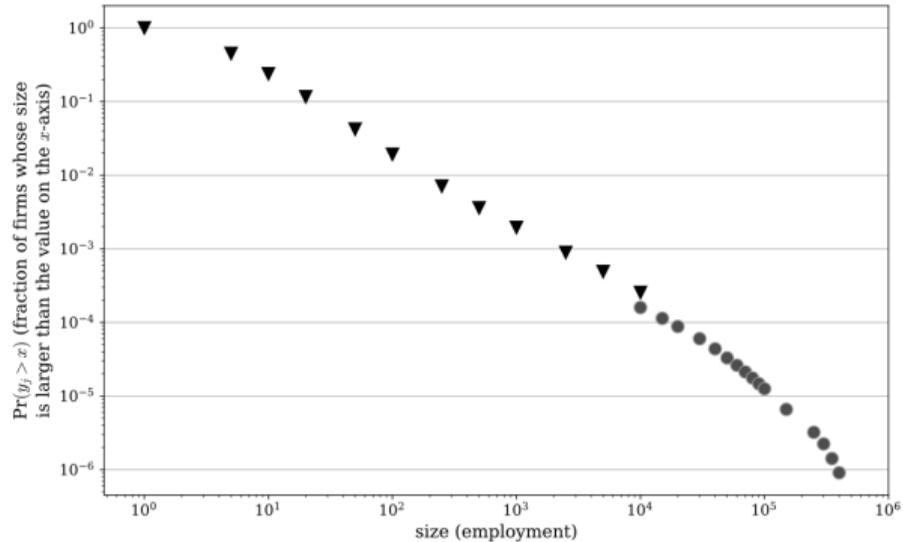- **Firm size** = number of product lines it owns.

Figure 22.9: Distribution of firm size using log-log axes.

**Source**: Business Dynamics Statistics and Compustat. Reproduced from Carvalho and Grassi (2019).

# Innovation & Growth

- **Limit pricing:** Leader charges $p = \lambda w$ to deter older-generation producers.
- Profit per product line: $\pi = (p - w) c$ with elasticity of substitution = 1.
- Incumbent invests in R&D to expand product lines; each line can be lost to competitor's innovation.
- Balanced-growth path: Growth rate $g$ proportional to economy-wide innovation rate $\mu$.

$$g = \mu \ln(\lambda).$$

# Firm Dynamics

- Each product line evolves via *Bernoulli trials* of "innovate and keep" vs. "lose to rival".
- In discrete time, we can track distribution of product lines across firms.
- Allows modeling of *entry/exit* and *endogenous* productivity process rather than exogenous $a_i$.

# Limitations & Extensions

- Baseline model: Growth of big firms is *negative on average* (contrasts with empirical patterns).
- Tail index vs. growth suggests modifications (e.g., cumulative innovation, different R&D structure).
- Despite simplicity, the Klette-Kortum framework unifies:
  - Innovation-based growth
  - Entry/exit
  - Firm size and dynamics

# Concluding Remarks

- Firm heterogeneity is critical to understanding macro outcomes, policy impacts, and productivity growth.

- Data show strong dispersion, reallocation, and evolving patterns (mega-firm rise, declining dynamism).

- Simple frameworks illustrate how dispersion affects aggregate productivity through allocation.

- General equilibrium models (e.g., (Hopenhayn and Rogerson, 1993)) show policy influences on hiring, firing, entry, exit.

- Endogenous growth perspectives (Klette and Kortum, 2004) connect innovation to firm turnover and macro growth.

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# Thank you!

Questions or comments?
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