Chapter 6: Welfare

Graduate Macroeconomics Slides

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January 23, 2026

Outline

- Introduction and Motivation

- The First Welfare Theorem
    - Statement and Proof
    - Tracing Out the Pareto Frontier

- Inefficient Market Outcomes
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    - Externalities
    - Missing Markets
    - Lack of Commitment
    - Market Power
    - Quantifying Welfare Losses

Outline (cont'd)

- Overlapping Generations (OG) Models
  - The Endowment Case
  - Intertemporal Production
- Optimal Government Policy
  - Missing Markets and the "Chicken Model"
  - Redistribution Policy
- Conclusion

Motivation

- This chapter explores when competitive equilibria lead to Pareto-optimal outcomes.
- The **First Welfare Theorem (FWT)**: under certain conditions, a competitive equilibrium is Pareto optimal.
- Markets coordinate actions of heterogeneous agents, often in a socially optimal way (the "invisible hand" intuition).
- In many macroeconomic settings, market failures can occur (e.g. externalities, taxes, missing markets).
- Understanding welfare implications helps us evaluate policy interventions.

Chapter Roadmap

1. Revisiting the **First Welfare Theorem**
2. Mapping the theorem into applied macroeconomic settings
3. Common market failures (taxes, externalities, missing markets, etc.)
4. Overlapping generations and potential inefficiencies
5. Implications for **government policy**

First Welfare Theorem: Statement

First Welfare Theorem (FWT): In an economy where

- Consumers and firms behave competitively (price-takers),
- Markets exist for all goods traded,
- Preferences satisfy *local non-satiation (LNS)*,

then a competitive equilibrium allocation is *Pareto optimal*.

Key idea: In equilibrium, no mutually beneficial rearrangement of resources remains unexploited.

Setup of the Abstract Economy

- A finite (or infinite) set of consumers $i \in I$; each with utility $u_i(x_i)$ and endowment c_i.
- A finite (or infinite) set of firms $j \in J$; each with production set Y_j.
- A price system $p \in \mathbb{R}^n$ (for n different goods).
- Competitive equilibrium:
  - Consumers maximize utility subject to budget constraints and $x_i \in X_i$,
  - Firms maximize profit $\pi_j = p \cdot y_j$ subject to $y_j \in Y_j$,
  - Market clearing: $\sum_i x_i \ \leq \ \sum_j y_j \ + \ \sum_i c_i$.

Sketch of the FWT Proof

Idea: Suppose a competitive equilibrium allocation (x_i^*, y_j^*, p^*) is *not* Pareto optimal. Then there is a *feasible* $(\widetilde{x}_i, \widetilde{y}_j)$ that Pareto dominates (x_i^*):

$$\widetilde{x}_i \succsim_i x_i^* \quad \forall i, \quad \widetilde{x}_{\hat{i}} \succ_{\hat{i}} x_{\hat{i}}^* \text{ for some } \hat{i}.$$

Under local non-satiation, each consumer's new consumption bundle cannot be *cheaper* than the old one at p^* (otherwise they'd have bought it before). Summing budget constraints and firm profit constraints leads to a contradiction with resource feasibility.

Conclusion: The assumption that (x_i^*, y_j^*, p^*) was not Pareto optimal must be false.

Intuition via Marginal Conditions

- In a competitive equilibrium, **all agents face the same prices**.
- For consumers, ratio of marginal utilities (MRS) equals price ratio.
- For profit-maximizing firms, ratio of marginal products (MRT) equals price ratio.
- $\underbrace{\text{MRS of consumer A} = \text{MRS of consumer B} = \dots}_{\text{common ratio}} = \text{MRT}.$
- No "marginal" improvement possible if everyone shares the same trade-off.

Applied Macroeconomic Settings

- In a **static** model with endowments or production, equilibrium satisfies MRS = MRT.
- In **dynamic** models (e.g. infinite horizon):

$$\text{Euler equation: } \beta u'(c_t) = p_t \quad \Leftrightarrow \quad \frac{u'(c_t)}{u'(c_{t+1})} = \frac{p_t}{p_{t+1}}.$$

- Firms rent capital at rate $r_t = \frac{p_t}{p_{t+1}} - 1$, so MRS = MRT across periods.
- **Key assumption:** Price-based mechanism is well-defined for all goods (including future ones); no externalities, no missing markets, etc.

Social Planner's Problem

Pareto frontier: set of all allocations that are Pareto efficient.

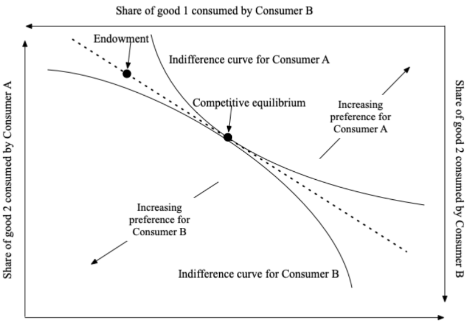
Method: Maximize a weighted sum of utilities ("Negishi weights"):

$$\max_{\{x_i\}} \sum_i \mu_i \, U_i(x_i),$$

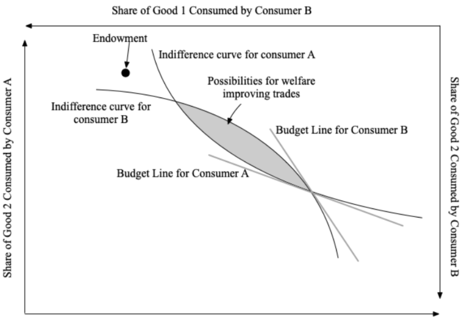
subject to feasibility: $\sum_i x_i \leq \sum_i c_i + \sum_j y_j$.

Implication:

- By varying μ_i, we get different Pareto-optimal points.
- There exists a distribution of initial wealth (endowments) s.t. the competitive equilibrium *coincides* with the social planner solution for a specific set $\{\mu_i\}$.
- Relates to the **Second Welfare Theorem**: any Pareto-optimal allocation can be implemented by a suitable redistribution of endowments and a competitive market.



(a) Efficient equilibrium.



(b) Distorted market.

- Edgeworth boxes depicting an endowment economy with two goods and two consumers. The top panel shows an efficient equilibrium in which both consumers face the same prices and have the same budget line. The lower panel shows a distorted market where the two consumers face different prices.

Negishi Weights & Competitive Equilibria

- Suppose each consumer i has Lagrange multiplier λ_i for budget.
- From consumer's FOC: $\beta^t u'(c_{i,t}) = \lambda_i p_t$.
- From planner's FOC: $\mu_i \beta^t u'(c_{i,t}) = \psi_t$ (the multiplier on aggregate feasibility).
- Matching terms suggests $\mu_i = 1/\lambda_i$ and $\psi_t = p_t$.
- $\Rightarrow$ **Competitive equilibrium** with some wealth distribution $\{\lambda_i\}$ is identical to **planner's solution** for weights $\{\mu_i\}$.

When the First Welfare Theorem Fails

Key assumptions behind the FWT can break down:

1. **Distortionary taxation** (consumers/firms face different after-tax prices).
2. **Externalities** (one agent's choice affects another's payoff without compensation).
3. **Missing markets or incomplete markets** (e.g. no insurance for certain risks).
4. **Lack of commitment** (inability to enforce future contracts).
5. **Market power** (monopoly or oligopoly).

When any of these arise, the equilibrium can be *Pareto-inefficient*.

Taxes: Lump-Sum vs. Distortionary

- **Lump-sum taxes:** Do not affect marginal decisions. Equivalent to a redistribution of endowments. Still no inefficiency introduced.
- **Distortionary taxes:** e.g. proportional tax on labor or capital income. These alter relative prices faced by consumers/firms:

$$w \to w(1 - \tau_\ell), \quad r \to r(1 - \tau_k).$$

- MRS $\neq$ MRT because after-tax wage or interest rate differs from the before-tax price.
- Summation argument in the FWT proof fails because consumer/firm see different effective prices.

Externalities: Example

Negative Production Externality:

$$y_j = A\left(\sum_{j'} y_{j'}\right) \cdot k_j^{\alpha} \ell_j^{1-\alpha},$$

where $A(\cdot)$ decreases in total output. Each firm ignores how its output reduces others' productivity.

Failure: FOC for each firm does not incorporate social cost. The competitive equilibrium yields too much production.

Interpretation: The TFP externality or pollution example. Pigouvian tax or property rights could restore efficiency.

Missing Markets: Borrowing Constraints

- Households may want to borrow (to smooth consumption) but face constraints $a_{t+1} \geq 0$.
- Then the standard Euler equation becomes an *inequality*:

$$\beta u'(c_{t+1}) \leq p_t / p_{t+1},$$

  with strict inequality if the constraint binds.
- Competitive equilibrium no longer achieves $\text{MRS}_{t,t+1} = \text{MRT}_{t,t+1}$ for constrained agents.
- Summation argument again fails: that alternative $(\tilde{x}_i)$ is *infeasible* for an individual due to the constraint, even if feasible in aggregate.

Lack of Commitment

- Agents or governments may be unable to commit to future actions.
- e.g. **Sovereign default** in international macro: can reneg on debt.
- e.g. Government may ex-post change policy (time inconsistency).
- Leads to incomplete contracts or ad-hoc constraints in equilibrium.
- Efficiency can fail because trades that would be mutually beneficial in a committed environment are blocked by fear of default/reneging.

Monopolistic Competition Example

Dixit-Stiglitz setup:

$$U = \left(\int_0^1 c(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad c(i) = \text{quantity of variety } i.$$

Each firm faces *downward-sloping demand* and charges a price $p(i)$ above marginal cost. Markup: $\mu = \frac{\varepsilon}{\varepsilon-1} > 1$.

Result: Output is lower than the competitive level; MRS $\neq$ MRT. Welfare is reduced by the markup wedge.

Measuring Welfare Differences

- Often we want a *consumption-equivalent variation*:

$$\Delta \text{ s.t. } u\big((1+\Delta)c^*, h^*\big) = u(c', h'),$$

- Interprets the utility difference in terms of a percentage of consumption needed to be as well off as in the alternative policy scenario.
- In dynamic models, Δ can be a permanent shift in consumption each period.
- Useful for policy comparisons (e.g. comparing lump-sum vs. labor-income taxation).

Overlapping Generations

- Multiple cohorts alive at once, each with finite lifetime.
- Even if markets are *complete* at any point in time, infinite horizon across cohorts can break the FWT.
- **Balasko-Shell theorem** (Balasko and Shell, 1980): an OG endowment economy can have *inefficient* equilibria if the present-value sum of endowments is infinite.
- A **pay-as-you-go social security** scheme can sometimes improve welfare for all generations.

Overlapping Generations (OLG) and Efficiency

- OLG models explore intergenerational trade and savings decisions.
- Even with no market frictions, distortionary taxes, or monopolies, equilibria in OLG models may be **Pareto-inefficient**.
- Efficiency test: A simple litmus test (introduced but not proven).
- Also includes models with demographic turnover (e.g., death and birth), where efficiency is not guaranteed.

The Endowment Case

- Two-period lived representative agents. Preferences:

$$u_t(c_y, c_o) = \log c_y + \log c_o$$

- Endowments: $\omega_{y,t} = \omega_y, \omega_{o,t} = \omega_o$
- Budget: $c_y + q_t c_o = \omega_y + q_t \omega_o$
- Competitive equilibrium (no trade):

$$c_{y,t} = \omega_y, \quad c_{o,t} = \omega_o, \quad q_t = \frac{\omega_y}{\omega_o}$$

Pareto Inefficiency and an Alternative Allocation

- Suppose $\omega_y = 3, \omega_o = 1 \Rightarrow q_t = 3$.
- Alternative allocation: $\tilde{c}_y = \tilde{c}_o = 2$
- All agents prefer $(2, 2)$ over $(3, 1)$: higher utility from equal consumption.
- But: Is $(2, 2)$ feasible for all generations?

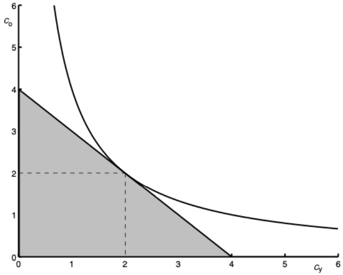


Figure: Figure 6.2: Pareto optimality of (2, 2) allocation

Balasko–Shell Theorem and Efficiency Criterion

- OLG equilibrium is Pareto optimal $\iff$

$$\sum_{t=0}^{\infty} \frac{1}{p_t} = \infty$$

- With $q_t = 3$, $p_t = 3^{-t} \Rightarrow$ finite sum $\Rightarrow$ inefficient.
- With $q_t = 1/3$, $p_t = 3^t \Rightarrow$ infinite sum $\Rightarrow$ efficient.
- Case $(\omega_y, \omega_o) = (2, 2)$ gives $q_t = 1$, equilibrium is efficient.

Transfer Sequences and Government Intervention

- With $(\omega_y, \omega_o) = (3, 1)$, achieving $(2, 2)$ requires a transfer chain.
- Feasibility fails: each generation needs more compensation than it gives.
- Government intervention: “Pay-as-you-go” can mimic intertemporal transfers.
- Only if transfer chain is sustainable does $(2, 2)$ become implementable.

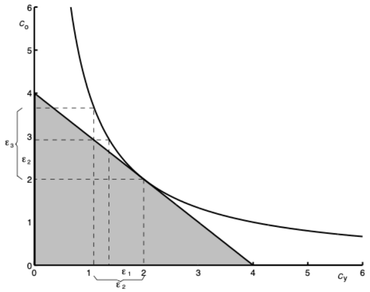


Figure: Figure 6.3: (2, 2) cannot be improved upon

Including Capital Accumulation

- In neoclassical OG production:

$$c_t + k_{t+1} = F(k_t, N_t) + (1-\delta)k_t,$$

- If net marginal product of capital $F_k(k^*) + 1 - \delta > 1$, we have *dynamic efficiency*.
- If $F_k(k^*) + 1 - \delta < 1$, we have *dynamic inefficiency*: cutting capital accumulation can raise consumption at all times.
- The condition is akin to $r^* \geq 0$ for dynamic efficiency.

Role of Policy

- **Ramsey Taxation:** Minimizing inefficiency given constraints on feasible tax instruments (capital/labor taxes, etc.).
- **Pigouvian Taxes/Subsidies:** (Pigou, 1920) Address externalities (pollution, knowledge spillovers).
- **Competition Policy:** Limit monopoly power or regulate natural monopolies.
- **Social Security (pay-as-you-go):** In overlapping-generations models with $r < 0$, such a transfer can improve welfare for all generations.
- **Caution – the "Chicken Model"**: If a market is *missing*, there may be reasons (information, enforcement). Government policy must solve or circumvent that root cause, not just fill in absent markets.

The "Chicken Model" Critique

- Suppose *markets do not provide* a certain good (like "chicken").
- Conclusion: Government should produce chicken.
- **But why are markets missing?** If it is due to moral hazard, enforcement, or asymmetric information, direct government provision might not solve it.
- **Lesson:** Identify the underlying friction. If government can fix that friction, a policy remedy can restore efficiency. Otherwise, policy might backfire.

Redistributive Concerns

- **Pareto efficiency** is silent on *equity*.
- **Social welfare functions** impose value judgments. For instance, *utilitarian*:

$$W = \sum_i U_i(x_i),$$

  often with *equal* weights across agents.
- Equilibrium might place high weight on those with large endowments. A policymaker might prefer a more equal distribution.
- “Veil of ignorance” justification: Agents agree on ex-ante insurance for uncertain endowments or abilities.

Summary

- Under ideal conditions, **competitive equilibrium** is *Pareto optimal*.
- When conditions fail (taxes, externalities, missing markets, etc.), the outcome is generally inefficient.
- In **overlapping-generations** models, even with no standard frictions, inefficiency can arise if the real interest rate is negative in the limit.
- **Policy** can sometimes restore or improve efficiency: Pigouvian taxes, competition policy, or pay-as-you-go transfers in OG settings.
- **Distributional issues** go beyond Pareto efficiency; *welfare weights* or the "veil of ignorance" principle can motivate policy for equity.

References I

Balasko, Y. and Shell, K. (1980). The overlapping-generations model. i. the case of pure exchange without money. *Journal of Economic Theory*, 23(3):281–306.

Pigou, A. C. (1920). *The economics of welfare*. Macmillan.

Appendix: Key Mathematical Details I

1. Proof Sketch for Overlapping Generations Dynamic Inefficiency

Consider the condition $f'(k^*) + 1 - \delta < 1$. We reduce k_{t+1} slightly, freeing resources in period t. Because $f'(k^*) < 1 - \delta$, we lose less than we gain overall. Repeating every period yields strictly higher $\{c_t\}$ for all t. Hence original equilibrium was inefficient.

Appendix: Key Mathematical Details II

2. Balasko-Shell Theorem (Informal)

- If $\sum_{t=0}^{\infty} 1/p_t < \infty$, the standard FWT summation argument fails.
- Then feasible redistributions across generations can lead to Pareto improvements.
- If $\sum_{t=0}^{\infty} 1/p_t = \infty$, no such chain of transfers can *strictly* improve welfare for all cohorts.

Thank you!

Questions or comments?

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